GUEST LECTURE 2: GAME THEORY & ITS APPLICATIONS

EXAMPLES & APPLICATIONS

Please note that if there are discrepancies between these lecture notes and those derived in class, these should be considered as correct. In fact, in class I may write notes, equations, etc. not because they are right, but to generate topics and discussions. In any case, you should always double check for yourself the correctness of the notes written in class and the notes attached here with the suggested textbooks or other means.

Recall that in the last class we motivated our conversation about competition structures and game theory by examining the continuum of competition, starting with monopoly and shifting towards perfect competition. Based on the simple linear model where *P* = *A* – *Q* and marginal cost is a constant *c* > 0, we got the below table:

Table 1: Competition structures when firms choose output

Item	**Monopoly**	**Duopoly**	**Oligopoly**	**Perfect competition**
Firms	1	2	N	N → ∞
q	(1/2)[A – c]	(1/3)[A – c]	(1/(N + 1))[A – c]	→ 0
Q	(1/2)[A – c]	(2/3)[A – c]	(N/(N + 1))/[A – c]	→ [A – c]
Price	(1/2)[A + c]	(1/3)[A + 2c]	(1/(N + 1))/[A + Nc]	→ c

The math and mechanics of these solutions were done by Cournot in the 1800s and relied only on simple calculus (taking first order conditions on maximisation problems). Under Cournot we imagined that firms choose quantity.

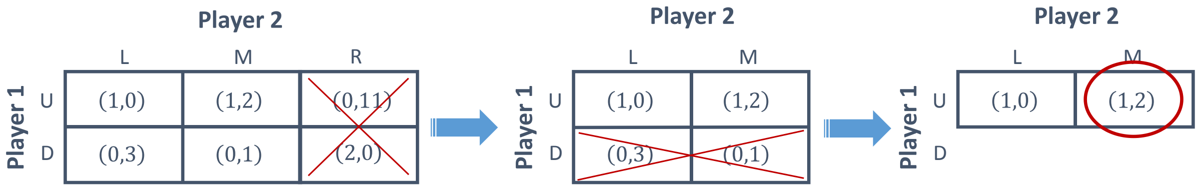
In the case of Bertrand it is assumed that firms compete on price. But under that assumption with just two firms in the market the outcome replicates the perfect competition model; i.e. price is equal to marginal cost.

The equilibria in either case were a set of stable strategies; i.e. no player had an incentive to deviate. Whether firms in reality compete on output or prices is complex. The truth is that for most firms and markets there is a bit of both. Agricultural markets and the oil markets tend to resemble more the Cournot model and oligopolies where outputs are differentiated are more likely to resemble the Bertrand model.

Game theory was first born through parlour games (i.e. trying to model these games as a set of strategies and interactions of rational players) and the works of Cournot and Bertrand. Modern game theory originated in the 1930s and its epicentre was (and continues to be) Princeton University and the adjacent Institute for Advanced Studies (IAS). It was also at Princeton that John Nash – whose letter of recommendation for Princeton from his professor at Carnegie Mellon was simply one sentence: “This man is a genius” – would revolutionise game theory through his work to establish conditions under which games would generate at least one stable outcome/equilibrium. This became known as the Nash Equilibrium (1951).

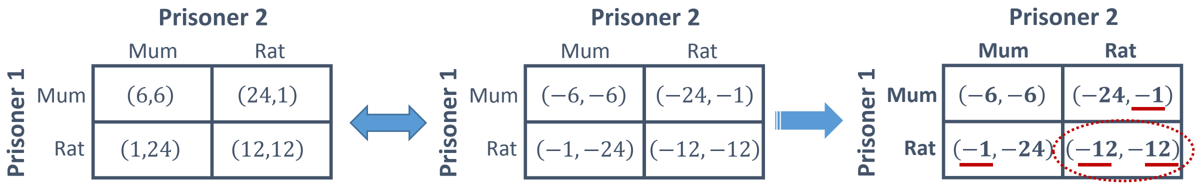
We can find the Nash Equilibrium of simple games by appealing to strictly dominant strategies, or their inverse, strictly dominated strategies. Recall that s_i' is strictly dominated by s_i'' if $u_i(s_i', s_{-i}) < u_i(s_i'', s_{-i})$. Rational players (and we will restrict ourselves to rational/sane behaviour for now) will not play strictly dominated strategies.

Recall that we also invoke “common knowledge” in games; i.e. I know that you know that I know… Under this guise we can eliminate strictly dominated strategies in some games to derive the equilibrium outcome. Consider the following example:



Thus by repeated iterations of eliminating strictly dominated strategies we can find the stable equilibrium outcome of a game.

Using the concept of dominance let's revisit the Prisoner's Dilemma. This time we will consider the game in normal utility form so that the payoffs are monotonic positive. This can be done with last week's example by taking the negative values of the matrix as jail time is a "bad" so the less negative the better for the prisoner.



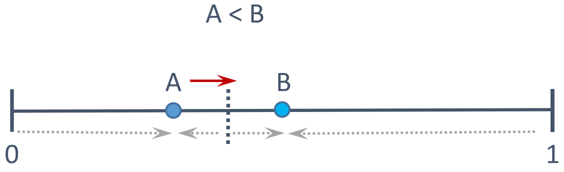
By underlining the strategy-payoff combination that each player undertakes given her rival's strategy we arrive at four marked numbers. If a cell in the matrix has all its elements marked then it is an equilibrium of the game.

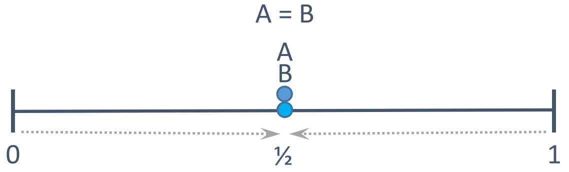
So there are two clever ways to get the equilibrium using the concept of dominance: (1) Eliminate strictly dominated strategies; (2) Find strategies that dominate others. Note that these techniques will not always find a solution. Indeed, often the solution is in mixed strategies. The interested reader should consult the suggested textbooks in the course outline on mixed strategies solutions.

HOTELLING (1929)

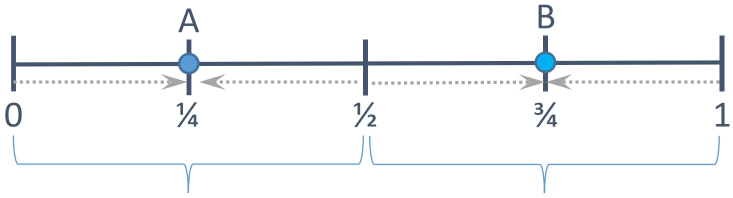
The Hotelling model of people distributed along a boardwalk is a clever and simple model that has many applications. It explains why the "centre" is usually crowded and has many fun and useful extensions. For example, we can add another axis which measures another variable (e.g. price). We can also conceptually think of it as a political spectrum to try to understand how voters choose candidates that are positioned on the political spectrum.

We said last week that along the [0,1] line under the game where people are spread uniformly and with just two vendors the equilibrium is for both vendors to position at the point ½. Graphically, we can see this is the case, because if it were not players would have an incentive to deviate.





If *A* < *B* then A has incentive to move to the right as *A*'s market share is [0, *A* + (1/2)(*B* – *A*)]. But the solution where both *A* and *B* are at point ½ is not efficient for consumers. Why? If consumers dislike walking there is an alternative distribution that involves less walking. In the {½, ½} solution the longest distance travelled by a consumer is ½. So now consider the solution {¼, ¾}. This, in fact, is the most efficient solution. Note here that the longest distance travelled by a consumer is ¼. The most correct way to demonstrate this efficiency is to calculate the total distance travelled by all consumers (do this a challenge/HWK for yourself), but here considering the longest distance travelled also works nicely. (Why?)



Challenge/HWK: Prove that the solution {1/3, 2/3} is not the most solution for consumers.

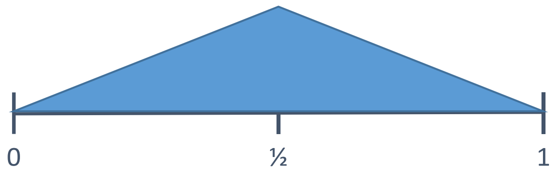
Challenge/HWK: Assume that the boardwalk is 100 metres long and there is one customer at 1 metre intervals starting from 0 to 100. People walk at the rate of 2m/s. Vendors can serve 1 customer every 3 seconds. People want to consume ice cream as quickly as possible. All consumers start game at same time and inelastically demand one ice cream cone and their own goal is to minimise time until they can consume the cone.

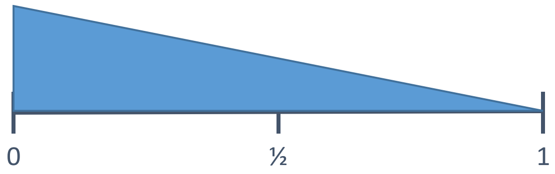
Question 1: If both vendors are required by law to sell the ice cream for the same price what is the solution? What if they can adjust their price?

Question 2: What if we assume that the vendors have different technologies? That is, suppose Vendor A can serve a customer every 3 seconds and Vendor B can serve a customer every 5 seconds. What is the solution then (if vendors are obligated to sell at the same price)?

Now suppose that we view Hotelling's model as a political spectrum story, but now assume that the distribution is not uniform.

Challenge/HWK: Assume that the Hotelling model applies over the unit interval [0,1] but that the distribution is not uniform. Namely that they take the forms below. What are the equilibria of the games below?





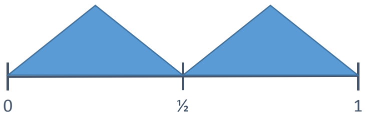
Hotelling's model is used extensively in political science to explain voting. This model is less good for Canada, where we have 3 or more major political parties (depending on the province and whether it's provincial versus federal). (Challenge to yourself: Show that there is no stable form solution with three players.) But in the United States where there are just 2 parties this should help in explaining their political dynamics. Then how come we do not see two centrist parties?

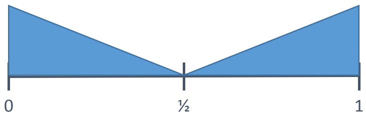
To perhaps help explain the dynamics of US politics (and generally also to be more realistic), suppose that consumers dislike walking and the utility of an ice cream cone is less than the disutility associated with walking more than ¼ units. This means that if a vendor is too far away then these people would just rather stay put than walk the distance for an ice cream. Under the politics analogy, voters will not bother to vote if there are no close candidates.

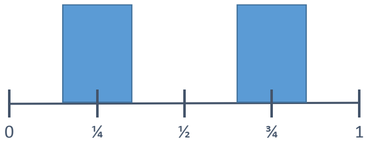
Then here we can generate a solution {¼, ¾}. This solution is more realistic in the sense that the parties are dissimilar (i.e. have different platforms) and is also efficient. This more closely resembles the current state of politics in the United States.

Challenge/HWK: Show that the above solution is an equilibrium.

Challenge/HWK: Recall that last week we wanted to find the equilibrium solutions for when the distribution of voters had the following shape:







BRANDER-SPENCER MODEL (1985)

Do government subsidies of industries make sense? Up until the 70s and the 80s the general consensus was no. Most models showed that government intervention usually resulted in inefficiencies and losses. However, when New Trade Theory (underpinned by economies of scale and geography) came about in that period it transformed how economists think about the role of government.

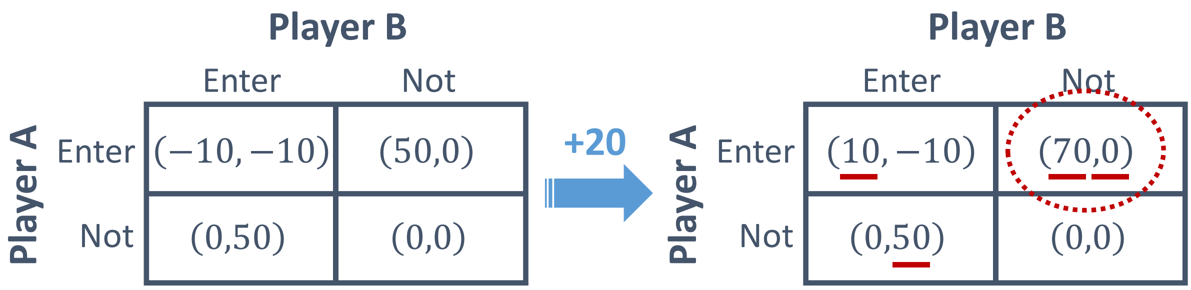
Two Canadian researchers (Bander and Spencer) from UBC came out with a simple stylised model to demonstrate when government subsidies may be optimal. Paul Krugman, the Nobel laureate and public intellectual, popularised a simplified version of this model in the 1990s.

Consider the below game. We call the two firms Player A and Player B. This naming convention is typically used because the closest real-world example of this scenario are that of rivals Airbus (Europe) and Boeing (USA). We imagine that the two firms are contemplating entering a third market. The market is such that if both enter they will both lose $10 billion. The motivation behind this logic is that there are large set-up/entry costs. Think of the R&D that is needed to develop aeroplanes. If one enters and the other stays out, then the active firm earns $50 billion. If both stay out they each earn zero. The below matrix summarises the game.

$$\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{\textbf{Player B}} \\
 & & \text{Enter} & \text{Not} \\ \hline
\textbf{Player A} & \text{Enter} & (-10,-10) & (50,0) \\ \hline
 & \text{Not} & (0,50) & (0,0) \\ \hline
\end{array}
\quad \Longrightarrow \quad
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{\textbf{Player B}} \\
 & & \text{Enter} & \text{Not} \\ \hline
\textbf{Player A} & \text{Enter} & (-10,-10) & (\underline{50},\underline{0}) \\ \hline
 & \text{Not} & (\underline{0},\underline{50}) & (0,0) \\ \hline
\end{array}$$

We can derive the Nash equilibrium solution of this game by examining the dominant strategies and noting that the strategies {(Not Enter, Enter), (Enter, Not Enter)} have all their payoffs underlined. However, this game has multiple equilibria and we cannot ascertain which equilibrium will prevail.

Now suppose that the government of Firm A can (credibly) commit to subsidise Player A if it enters the market. Thus the new game becomes:



Player B

Player A	Enter	Not
Enter	$(-10,-10)$	$(50{,}0)$
Not	$(0{,}50)$	$(0{,}0)$

Player B

Player A	Enter	Not
Enter	$(\underline{10},-10)$	$(\underline{70},\underline{0})$
Not	$(0,\underline{50})$	$(0{,}0)$

This policy by the government is welfare improving. It generates a \$50 billion benefit to the country net of its cost of \$20 billion. Previously it was uncertain which country would enjoy the \$50 billion gain. In fact, the expected gain in that case was \$25 billion. Thus, ignoring issues of the costs of taxation and transferring, and also ignoring issues of distribution, the move is a \$25 billion net positive investment for the country (in expected value).

BRANDER-SPENCER MODEL AS STACKELBERG GAME

Recall with two players that $Q \equiv q_1 + q_2 \Rightarrow P = A - (q_1 + q_2)$. Then

$$\pi_A = q_A p(Q) - c(q_A) + s q_A \text{ and } \pi_B = q_b p(Q) - c(q_B)$$

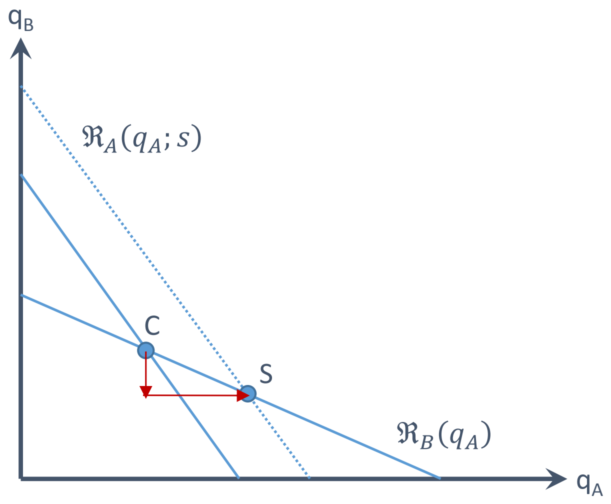
where *s* > 0 is a subsidy provided to Firm A (but not to Firm B). Each firm takes the other's choice of quantity as given, so its F.O.C.s are:

$$A\colon q_A \frac{\partial p(Q)}{\partial q_A} + p - c'(q_A) + s = 0$$

and

$$B\colon q_B \frac{\partial p(Q)}{\partial q_B} + p - c'^{(q_B)} = 0 \Rightarrow \; q_B^* = \frac{c\prime(q_B) - p}{p\prime(Q)}$$

These yield the reaction functions for A and B: $\Re_A(q_A; s)$ and $\Re_B(q_A)$ which are sketched below.



The subsidy has an effect similar to the Stackelberg first-mover advantage. It effectively shifts the reaction function of Firm A by the subsidy amount. So the equilibrium moves from the Cournot solution (at point *C*) to the Stackelberg solution (at point *S*). Here, Firm A produces more output than the Cournot solution while Firm B reduces its output from the Cournot solution.

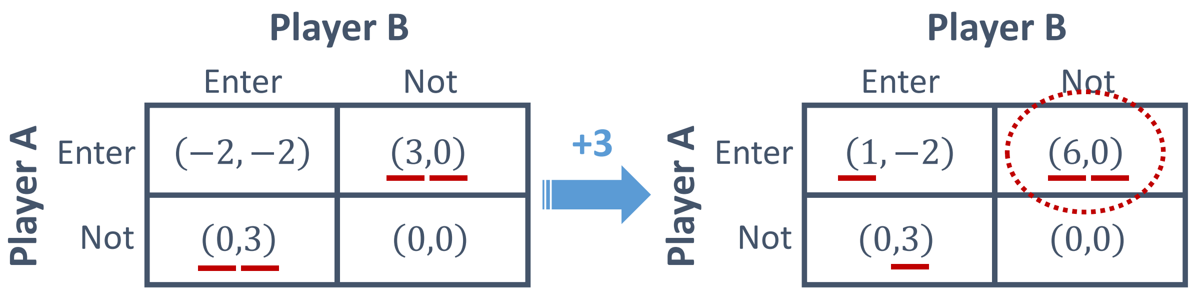
EXAMPLE 2 WITH BRANDER-SPENCER

Assume that costs to design and develop (R&D) aircraft are $6 billion. Thereafter, planes can be produced at a constant cost of $5 million per plane. The demand in the (third) market is such that if both firms enter they are able to each sell 400 units at $15 million per plane. However, if just one firm enters it enjoys monopoly status and will sell 600 units at $20 million per plane.

In the above scenario, the firms experience a loss of $2 billion if both enter, but a profit of $3 billion if just one of them enters.

Challenge/HWK: What parameters on the simple linear model ($P = A - Q$ and $MC = c$) yields a monopoly solution of 600 outputs and duopoly total outputs of 800?

The game above can be summarised in the normal form matrix below:



The game without government intervention boils down to a game of chicken/dare, where each opponent stares at each other and the first to flinch (i.e. decided to play strategy Not Enter) loses. So the subsidy in effect gives the first-mover advantage to the firm. When the government can credibly commit to irreversibly subsidise the R&D costs (or some portion thereof) then this deters the rival firm from entering the market.

In Quebec there are many tax breaks for firms that engage in innovation and in particular R&D. In fact, one of the most popular programmes (SRED) allows firms to effectively claim 50 percent of their R&D costs. If the firms above were given such a subsidy this would imply that the government would rebate them $3 billion in their initial cost. Thus a firm’s net operating profit after R&D and rebate would be $1 billion.

But if both government think and act alike (and hence subsidise their firm) then we arrive at the Prisoner’s Dilemma outcome. The difference then is that previously (when the firms both entered) the losses were private. Now in the case of a double subsidy, the loss is incurred by the government, i.e. taxpayers bear the cost, while the firms enjoy a profit.

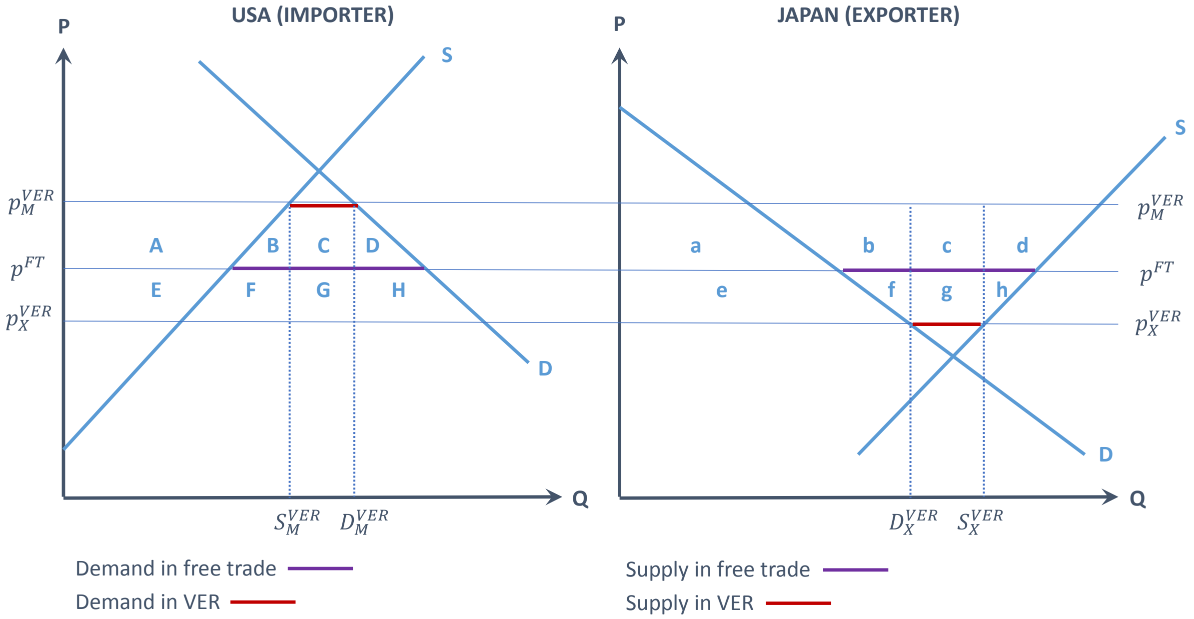
In a political economy framework we are likely to see the case of the dual subsidy leading to private profits and public losses. This is because the gain to the firms is significant, whereas the cost to taxpayers – even though they outweigh the gain to the firm – are spread over a large base. This is exactly the issue with “special interest”. Whereas each taxpayer may need to pay an extra $50 in taxes, the firm will earn $1 billion (in the example where the government offers a 50 percent rebate on R&D). Thus individual taxpayers are not likely to lobby the government and fight against the subsidy; whereas the firm with $1 billion to make will hire lobbyists, make campaign contributions to politicians, etc.

So the moral of the story is that the real world is complex. At first we thought we found a clever solution to support the domestic firm and generate welfare gains for the country through a subsidy programme. But then we realised that the problem is symmetrical for the foreign firm and it too may lobby the government – under the same arguments – for a subsidy to help capture the market and welfare gains. When this happens both firms enter and lose, but their losses are offset by the subsidy. So private gains are funded by public losses. Based on the realities of political economy, we are likely to see such a scenario as the incentives are for the firms to lobby for the subsidy as there is little incentive for individual taxpayers to fight back.

VOLUNTARY EXPORT RESTRAINT

A voluntary export restraint (VER) is a government imposed limit on the quantity of some goods that can be exported to a specific country. VERs arise when domestic producers seek protection from foreign producers. VERs are “offered” by exporting countries.

Case study: Japanese automobile manufacturer VERs in the 1980s in the US market.



The welfare impact (recall the notes on how to interpret consumer and producer surplus) is captured in the table below.

Table 2: Welfare effects of a VER

	USA (importer)	Japan (exporter)
Consumer surplus	– (*A* + *B* + *C* + *D*)	+ *e*
Producer surplus	+ *A*	– (*e* + *f* + *g* + *h*)
Quota rents	0	+ (*c* + *g*)
National welfare	– (*B* + *C* + *D*)	*c* – (*f* + *h*)
World welfare	– (*B* + *D*) – (*f* + *h*)	

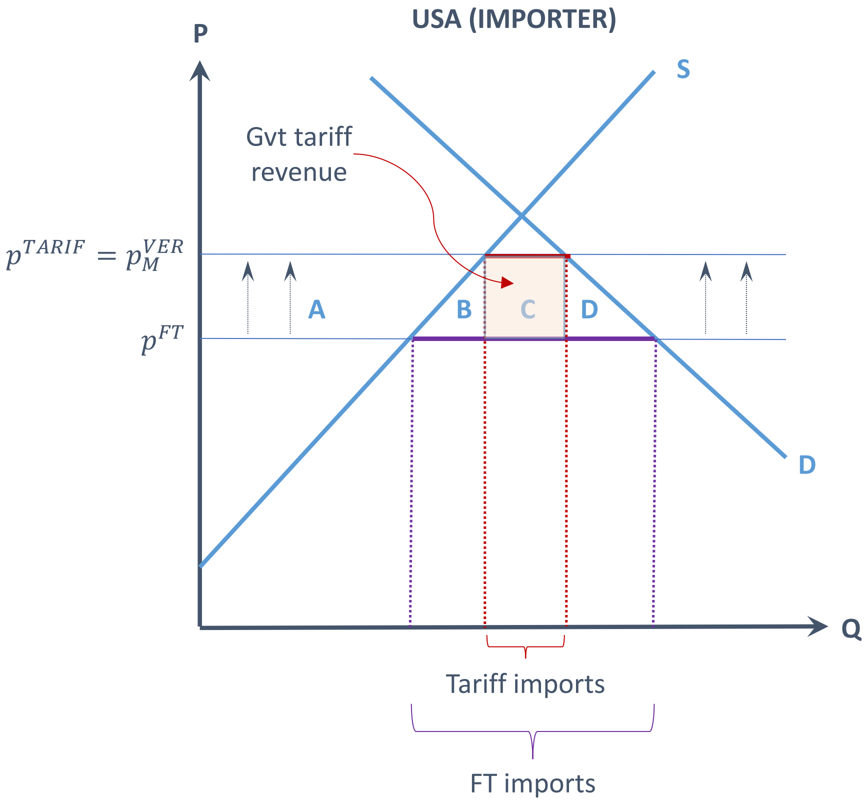
So US automakers gain *A* through the VER while Japanese firms lose (*e* + *f* + *g* + *h*). So in that sense the US government's request that Japan enact a VER did indeed help US car makers. However, there was a significant cost imposed on US consumers who end up paying more for cars. Conversely, Japanese consumers ended up better off as the price for cars in Japan decreased as output that was previously exported stayed in the Japanese market thus lowering prices in Japan.

Quota rents went to Japan. This is the value of the cars sold to USA at the VER price, less the price that prevailed in Japan.

Welfare in the US is unambiguously lower as consumers (large base) must pay more while producers (small base) gain *A*. The case for Japan, however, is ambiguous. If *c* is large then it can offset (*f* + *h*), which are the "deadweight triangles" associated with the VER. How big is *c*? This depends on the size of each country. If both the importer and exporter are large (i.e. they can influence market prices), then *c* is likely to be big and the national welfare impact to the exporter is likely positive. This was the case with USA-Japan with respect to the car market.

The moral of the story here is that the US government undertook bad policy in protecting their automakers. Their action essentially resulted in a gift to the Japanese. Although the policy may appear effective, a bit of analysis shows that it was a policy blunder. Nevertheless, the US government was limited by having reduced tools in its kit as GATT (the precursor to the WTO) had restrictions on the types of trade policies member countries could apply against others.

Challenge/HWK: If the US government were instead able to impose an *equivalent tariff*, what would be the welfare effects?



ADDENDUM

Challenge/HWK: Consider the game where two players each call out a number between 0 and 100. If the sum of the two numbers exceed 100 then both get zero. If, however, the sum does not exceed 100 then each gets the number they choose (in dollars). That is, the payoff matrix is:

		Player 2		
		$y \leq 50$	$y > 50$ $x + y \leq 100$	$y > 50$ $x + y > 100$
Player 1	$x \leq 50$	(x, y)	(x, y)	$(0,0)$
	$x > 50$ $x + y \leq 100$	(x, y)	▨	▨
	$x > 50$ $x + y > 100$	$(0,0)$	▨	$(0,0)$

What is the Nash Equilibrium of the game? What if instead the rule is changed so that if the sum exceeds 100 then the player with the smaller number gets that amount (in dollars) while the other gets zero?

Challenge/HWK: Another popular game is “take it or leave it”. In this game there are 2 players with the objective of splitting $100. Player 1 chooses a number between 0 and 100. Thus player 2 gest the remainder: 100 – x. The rule is if player 2 accepts the remainder then both keep the money; but if player 2 refuses then both get zero. What is the Nash Equilibrium of this game?

In the above, standard economic theory says that player 2 should accept any amount so long as 100 – x is greater than zero. However, when this game is conducted there are interesting results that show people have an intrinsic sense of fairness. Moreover, the game has very different results if the players have direct contact with each other or anonymously play the game (i.e. the players do not interact in person with each other). Namely, when the first player picks a large number so that 100 – x is much less than 50 then player 2 is highly likely to refuse. Moreover, when the players interact with each other directly the first player is more likely to pick a number close to 50, whereas when it is anonymous player 1 is more likely to choose a number much closer to 100. Can you explain why we observe these phenomena?

EQUIVALENT TARIFF

Let’s consider how a tariff works. A tariff is essentially a tax on imported goods. Depending on the market structure, it could result in a leftward shift of the supply curve in the importing country. Or we can imagine that a flat world supply curve is shifted up by the tax amount. In either case the quantity consumed at home decreases and imports decrease. The welfare effects are such that there will be a deadweight loss as the tariff introduces a friction into the market.