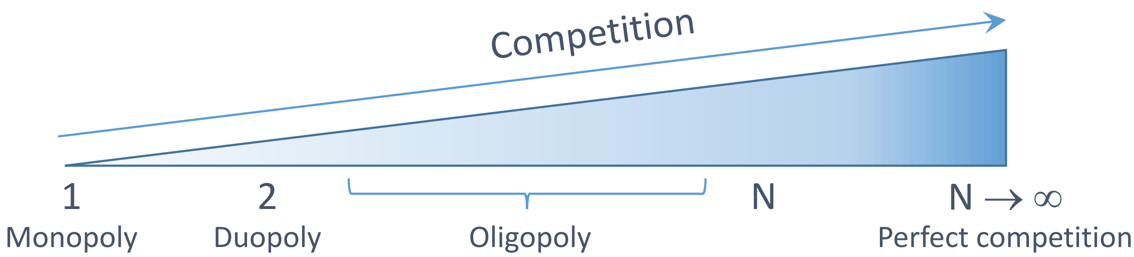
GUEST LECTURE 1: GAME THEORY & ITS APPLICATIONS

COMPETITION & COMPETITIVE STRUCTURES

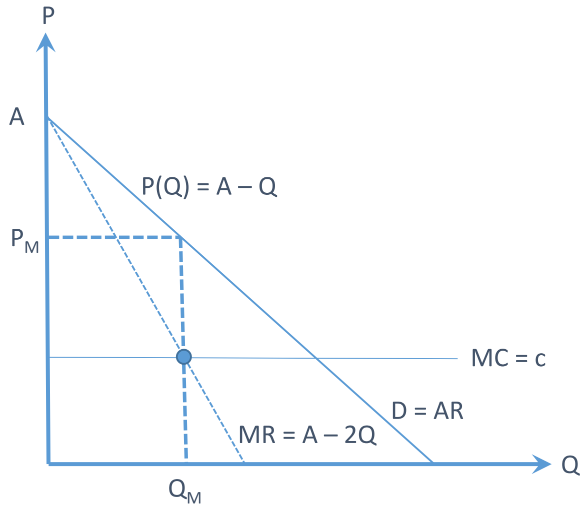
Please note that if there are discrepancies between these lecture notes and those derived in class, these should be considered as correct. In fact, in class I may write notes, equations, etc. not because they are right, but to generate topics and discussions. In any case, you should always double check for yourself the correctness of the notes written in class and the notes attached here with the suggested textbooks or other means.

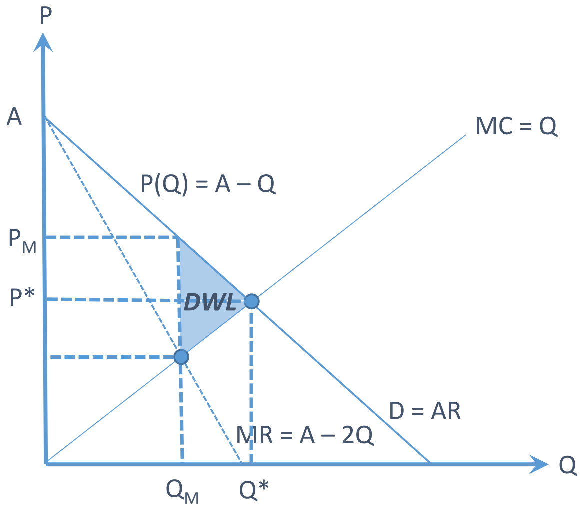
We will examine competition and competitive structures as a means of motivating discussions about game theory and, in particular, the phenomenon of Nash Equilibria in games.

Competition: Monopoly ➔ Duopoly ➔ Oligopoly ➔ N-firm structure ➔ Perfect competition



We will primarily focus our analysis with a simple linear demand function: $P = A - Q$.





Definition: A *monopolist* is a firm that has no similar competitors; it ignores potential reactions of other firms when choosing its output/price/strategy

In the following we assume no price discrimination; i.e. a firm must sell all units in the market at the same price.

In economics we assume (for better or worse) that agents are profit (or utility) maximisers.

We use the Greek letter π to denote profit. Thus a profit-maximising monopolist faces the following problem:

$$\max_{Q \in \mathbb{R}^+} \pi(Q) = R(Q) - C(Q)$$

The F.O.C. for this solution is then

$$R'(Q) - C'(Q) = 0 \Leftrightarrow MR(Q) = MC(Q)$$

We should technically also check that the S.O.C. is satisfied.

Definition: *Consumer surplus* is the difference between the maximum price a consumer is willing to pay versus the price paid. In the supply-demand chart this is the area between the demand curve and the price that a product is sold for.

Definition: *Producers surplus* is the difference between the price a product sells at and the cost of its production. In the supply-demand chart this is the area between the supply curve and the price that a product is sold for.

So monopolies are inefficient since as there are gains from trade that are not realised; i.e. there are consumers with a willingness to pay higher than the cost of production for the firm who are not served.

In competitive market p = MC ➔ all gains from trade are realised (efficient). But note that in the case where a monopolist can perfectly price discriminate and all the surplus goes to the firm is also an efficient outcome: Efficient ≠ Fair.

GAME THEORY

We shall restrict ourselves to one-off games of the form below:

1. Players simultaneously choose actions (strategies); then players receive payoffs
2. Players have common knowledge of all the strategies and payoffs available to them and to their rivals
3. Players obtain utility from the payoffs that satisfy the standard assumptions of utility theory

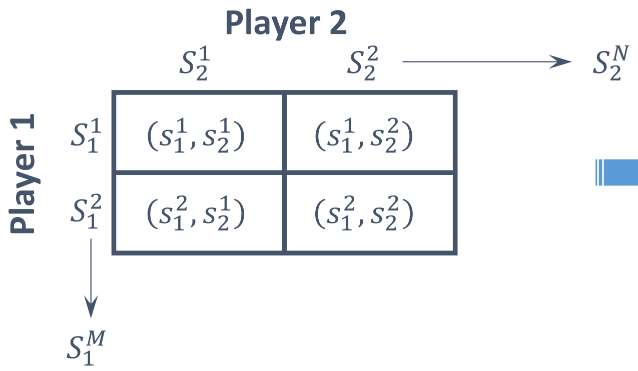
Repeated games are covered in the recommended textbooks. They are not required to derive a general understanding of a Nash Equilibrium, which is the ultimate goal behind these lecture notes.

Normal form game specifies the following structures of a game:

1. The players of a game
2. Ste of all available strategies
3. Payoffs for all strategies

Two-player normal form games can be represented in a matrix where each of the cells of the matrix list the payoffs to the players for a given set of strategies.

Example: Prisoner's Dilemma



Prisoner 1 \ Prisoner 2	Mum	Rat
Mum	(6,6)	(24,1)
Rat	(1,24)	(12,12)

The story behind this game is that each prisoner is held separately and the police do not have sufficient evidence of the larger crime they committed. But they can convict each on minor crimes. However, if one of the prisoners cooperates and helps convict the other on the larger charge the police will give the one that rats a lighter sentence while being especially harsh to the one that does not cooperate. If both Rat then both are fully convicted of the original big crimes.

The optimal strategy is for each prisoner to choose Rat. We call this the Prisoner's Dilemma since the prisoners would be better off if each could commit to play Mum. But Rat is an optimal strategy in that no matter what action the Player 2 takes, Player 1 is better off playing Rat.

Many real world scenarios in economics, politics, etc. highly resemble the Prisoner's Dilemma situation. That is, players would be globally better to cooperate, but given the situation players have an incentive to cheat/deviate from the cooperative solution. For example, OPEC and trade wars display such dynamics. Because players would be better off cooperating sometimes we see them entering into contracts to force each other to the good solution and they put in structures in place such that the good solution becomes an equilibrium (i.e. players do not want to deviate). For example (and especially in repeated games) the player that Rats will have his kneecap broken by an enforcement mechanism.

Definition (*Nash Equilibrium* (1951)): In the *N*-player game $G = \{S_1, \cdots, S_N; u_1, \cdots, u_N\}$ the strategies $(s_1^*, \cdots, s_N^*)$ are a Nash Equilibrium if $\forall i \in I$, s_i^* is player *i*'s best response to the strategies of the other N-1 players:

$$(s_1^*, \cdots, s_{i-1}^*, s_{i+1}^*, \cdots, s_N^*) \Leftrightarrow u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \forall s_i \in S_i$$

$$\Leftrightarrow \max_{s_i \in S_i} u_i(s_i, s_{-i}^*)$$

Nash's theory (which was his doctoral dissertation at Princeton) guarantees that an equilibrium (possibly in mixed strategies[1]) exists in a broad class of games (i.e. finite players with finite payoffs).

Definition (*Strictly Dominated*)**:** Strategy s_i' is strictly dominated by strategy s_i'' if

$$u_i(s_i', s_{-i}) < u_i(s_i'', s_{-i}) \forall s_i \in S_i$$

That is, player *i* (if she is sane/normal/rational) will never play s_i'.

[1] Mixed strategies are those actions that are played over a probabilistic distribution.

Conversely, a strategy strictly dominates an alternative strategy if the inverse of the above is true. We can use the concept of dominance (in both directions) to derive the Nash Equilibrium in some normal-form games.

BACK TO COMPETITION STRUCTURES

Now consider the duopoly case with $Q \equiv q_1 + q_2$ where q_i is output chosen by *i*.

Define the best response function as player *i*'s best strategy given its rival's strategy:

$$BR_i(\cdot) \equiv \Re_i = \max_{q_i \in \mathbb{R}^+} \pi_i(q_i, q_j)$$

So what is the best response function of *i*?

$$\max_{q_i \in \mathbb{R}^+} \pi_i(q_i, q_j^*) = \max_{q_i \in \mathbb{R}^+} q_i\left[A - (q_i + q_j^*) - c\right]$$

The F.O.C. is:

$$A - 2q_i - q_j^* - c = 0 \Rightarrow q_i^* = \frac{1}{2}(A - c - q_j^*)$$

The solution for *j* is symmetric, so we can solve:

$$q_i^* = q_j^* = \frac{1}{3}(A - c) \Rightarrow Q = \frac{2}{3}(A - c)$$

Compare this with the monopoly solution we saw earlier:

$$\frac{1}{3}(A - c) = q_i^* = q_j^* < Q^M = \frac{1}{2}(A - c) < Q^D = \frac{2}{3}(A - c)$$

That is, the duopoly total solution produces more than the monopoly.

Challenge/HWK: Prove that a duopoly that colludes to produce the monopoly output (i.e. each produces ¼(A – c)) is not an equilibrium outcome.

N-FIRM OLIGOPOLY

Define $Q \equiv \sum_{k=1}^{N} q_k$ and $q_{-i} \equiv Q - q_i = \sum_{k \neq i} q_k$

We assume all firms are identical and that $c_i = c \; \forall i$. Thus

$$\pi_i(q_i | q_{-i}) = p(Q)q_i - cq_i$$

The F.O.C. is

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\partial p}{\partial q_i} \times q_i + p(Q) - c = 0$$

$$\Rightarrow (-1)q_i + A - q_i - q_{-i} - c = 0$$

$$\Rightarrow \Re_i(q_i | q_{-i}) = \frac{1}{2}(A - c) - \frac{1}{2}q_{-i}$$

We thus have *N* symmetric equations of the form $-q_i + (A - Q) - c = 0$. Thus summing those *N* equations we get $-Q + N(A - Q) - Nc = 0$. So total output is: $Q = \frac{N}{N+1}(A - c)$. Note that as $N \to \infty$ then $Q \to A - c \Rightarrow p \to MC$. Moreover, $q_i = \frac{1}{N+1}(A - c) \to 0$ as $N \to \infty$. That is, as *N* gets large we approach the perfect model solution, as it should be.

BERTRAND MODEL OF COMPETITION

Previously we assumed firms choose output (and price is determined by the market based on the prevailing demand curve). Now suppose firms choose price. Does this yield a big change in the result? Yes! In fact, as we shall see, we can derive the infinite firm solution (i.e. perfect competition) with just two firms if we imagine that they choose price.

We will consider the case of Bertrand competition when firms produce homogeneous goods.

When firms are identical in every way (selling same good, same time, same distance from consumer, same…), then we can imagine the following demand function based on price:

$$d_i = \begin{cases} A - p_i & p_i < p_j \\ 0 & p_i > p_j \\ \frac{1}{2}(A - p_i) & p_i = p_j \end{cases}$$

where $min\{p_i, p_j\} \geq c$.

Conjecture: $p_1 = p_2 = c$ is the unique solution.

That is, we achieve the perfect competition outcome (p = MC) with just two firms when they compete on prices.

Proof (by contradiction): To prove the above, let's assume the opposite and show that it cannot be the case. For that we assume the following three cases and show that they cannot hold:

i. $p_1^* > p_2^* > c$
ii. $p_1^* = p_2^* > c$
iii. $p_1^* > p_2^* = c$

Starting with the first case:

$$p_1^* > p_2^* > c \Rightarrow \begin{cases} d_1 = 0 & \Rightarrow \qquad \pi_1 = 0 \\ d_2 = A - p_2 & \Rightarrow \pi_2 = d(p_2)(p_2 - c) > 0 \end{cases}$$

Since the real numbers are dense, this implies there exists $\varepsilon > 0$ such that Player 1 would prefer to set a price $p_1 = p_2 - \varepsilon$ for some $\varepsilon > 0$ sufficiently small.

Now consider the second case.

$$p_1^* = p_2^* > c \Rightarrow \begin{cases} d_1 = \frac{1}{2}(A - p_1) & \Rightarrow \pi_1 = (p_1^* - c)\frac{1}{2}d(p_1) > 0 \\ d_2 = \frac{1}{2}(A - p_2) & \Rightarrow \pi_2 = (p_{12}^* - c)\frac{1}{2}d(p_2) > 0 \end{cases}$$

But since the real numbers are dense, this implies there exists $\varepsilon > 0$ such that Firm 1's best response is $\Re(p_1^*) = p_2 - \varepsilon$.

Finally,

$$p_1^* > p_2^* = c \Rightarrow \begin{cases} \pi_1 = 0 \\ \pi_2 = (p_2^* - c)d(p_2^*) = 0 \end{cases}$$

Since the real numbers are dense this implies there exists $\varepsilon > 0$ such that the best response of Firm 2 is $p_2^* = p_1 - \varepsilon$.

Challenge/HWK: Revisit the below, but derive the results when firms have differentiated products. In particular, assume that the rival firm has a good whose substitutability can be parameterised by *b*. For this, what condition do we need to impose on *b* for this problem to make sense? For clarity, assume in this case that for differentiated goods we have $q_i(p_i, p_j) = d_i(\cdot) = A - p_i + bp_j$

STACKELBERG DUOPOLY

Hitherto we have assumed that firms choose their output simultaneously. Now introduce a game where one firm chooses a quantity first and then the second firm, after observing the choice of output by the first, responds accordingly. Otherwise, we keep all the prior assumptions.

To examine this case, we can without loss of generality assume that Firm 1 moves first.

Again firms choose quantity: $Q \equiv q_1 + q_2 \Rightarrow P = A - (q_1 + q_2)$.

To solve this we will use backwards induction. That is, at time *t* = 2 we have Firm 2 that solves the problem:

$$\max_{q_2 \in \mathbb{R}^+} \pi_2 = pq_2 - cq_2 = [A - (q_1 + q_2) - c]q_2$$

The F.O.C. is

$$\frac{\partial \pi_2}{\partial q_2} = 0 \Leftrightarrow A - q_1 - 2q_2 - c = 0$$

$$\Rightarrow q_2^* = \Re_2(q_1) = \frac{1}{2}(A - c - q_1)$$

Now go to period 1 and note that Firm 1 chooses its quantity knowing that Firm 2 will react according to it reaction function. Thus Firm 1's problem is:

$$\max_{q_1 \in \mathbb{R}^+} \pi_1 = pq_1 - cq_1 = [A - (q_1 + q_2^*) - c]q_1$$

Now plug in $q_2^* == \frac{1}{2}(A - c - q_1)$ and take the F.O.C.

$$\frac{1}{2}A - q_1 - \frac{1}{2}c = 0 \Rightarrow q_1^* = \frac{1}{2}(A - c)$$

Now solve for $q_2 = \frac{1}{2}(A - q_1 - c) = \frac{1}{4}(A - c)$. This implies that $q_1^* > q_2^*$ and $Q = \frac{3}{4}(A - c)$ and so $p^* = \frac{1}{4}(A + 3c) > c$ (since *A* > *c*).

HOTELLING (1929) GAME

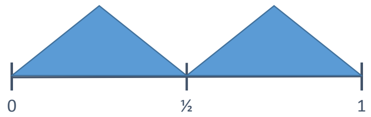
Consider a situation in which there is a 1-km long boardwalk along a beach with people uniformly spread across this stretch. It is a hot sunny day and all the people on the beach inelastically demand one ice cream cone. Two vendors must choose (simultaneously) where to locate themselves along the boardwalk. People patronise the vendor closest to them. There is no cost of travelling or queuing. The vendors sell identical goods and are identical in every way, except possibly in where they decide to set up their stand.

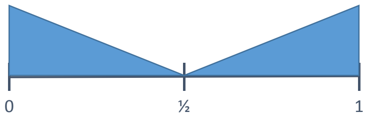
Question 1: What is the Nash Equilibrium for this game?

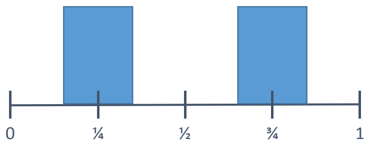
Question 2: Is this the most efficient location? (Recall: Efficient ≠ Fair.) What is the most efficient distribution if we are trying to maximise consumer welfare if the burden to consumers is the distance they have to walk to get their ice cream?

Question 3: What happens if the distribution of people is not uniform?

Challenge/HWK: Suppose that the distribution of the people on the beach takes on the following three shapes below. Can you describe the equilibrium?







Question 4: If instead we have three vendors, how does the result change?