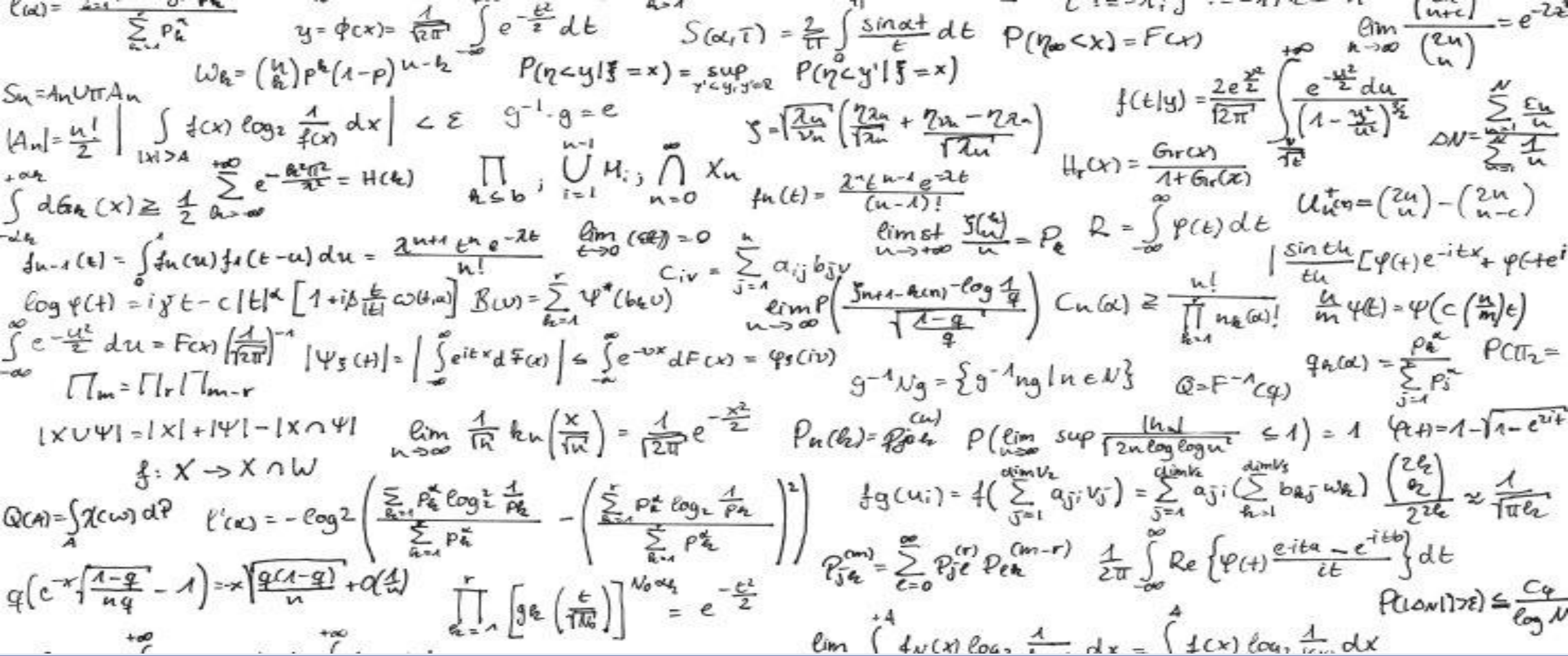

INSE 6441: Game theory guest lecture

Applications of game theory in business, finance and policy

INSE 6441: Game theory guest lecture (Concordia University)

21 Mar 2017

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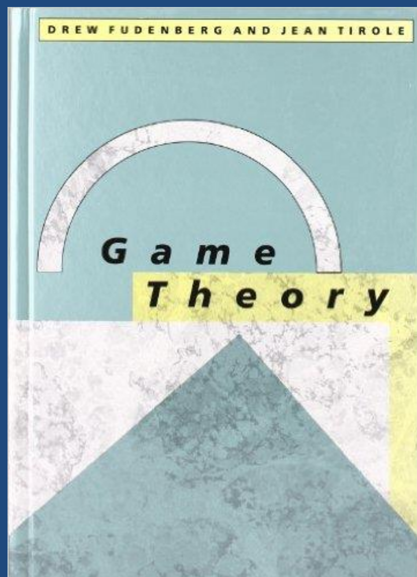
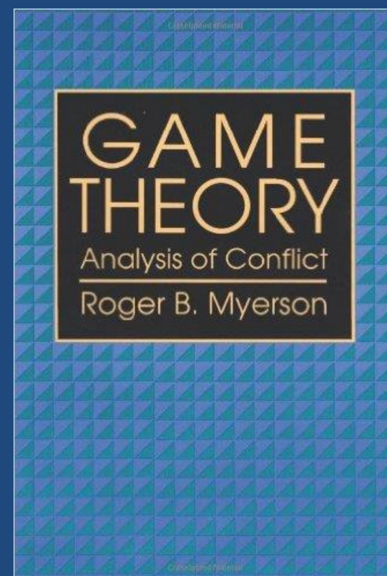
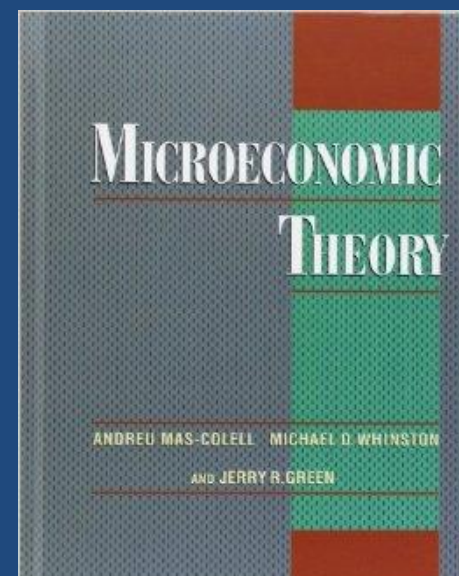
What is game theory?

A branch of economics that models interactions amongst agents



- The process of modelling strategic interactions between two or more players within a competitive situation
- The theory of social situations and how agents behave in relation to the actions of others



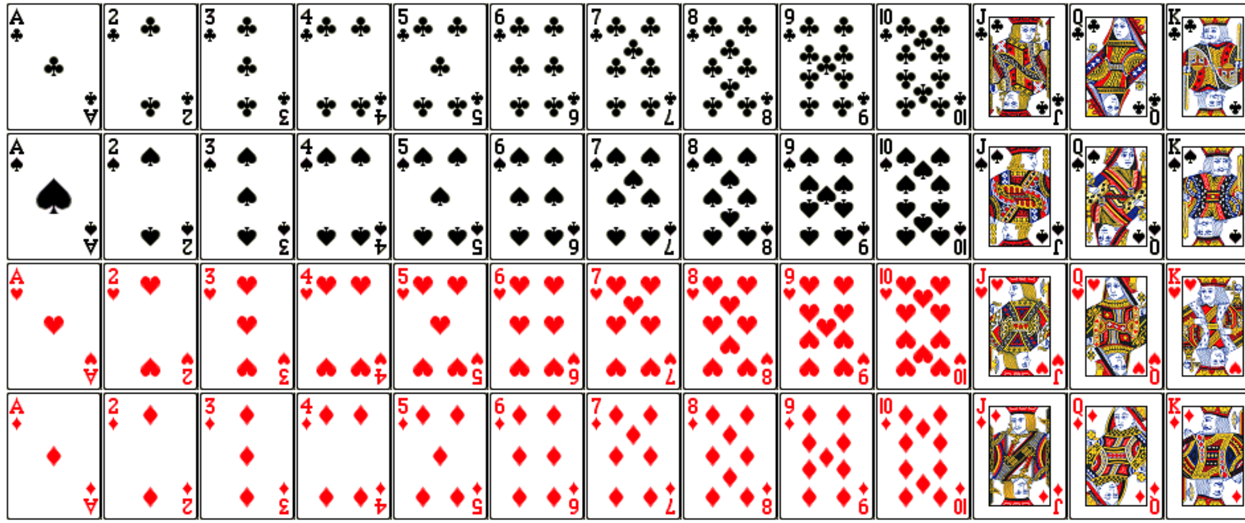


- Baseball: Batter can hit ball if he can anticipate pitch; otherwise, he will swing and miss…
- Hockey: Netminder can make save if he can anticipate player shooting or deking, else he will let in goal…

History of game theory

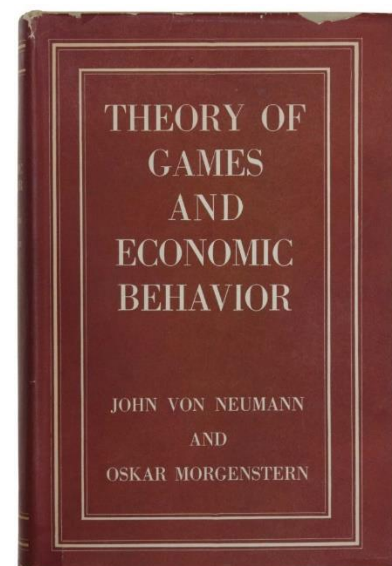
It all started with a card game…

Games such as cards (which is what supposedly launched formal game theory), chess and games of chance spurred the initial investigation into games.



Antoine Augustin Cournot (1801-1877) modelled the firms competing in duopoly market:

1. No collusion
2. Choose output
3. Simultaneous
4. Act rationally
5. Identical goods



Modern game theory

Standing on the shoulders of giants (Nobel Prize* winners)

von Neumann, Morgenstern, Selten, Nash, Harsanyi, Schelling, Aumann, Huwicz, Maskin, Myerson, Roth, Shapley, Tirole

Cournot vs Bertrand models of duopoly

Choosing quantity vs price in duopoly competition

$$P(Q) = a - bQ \text{ and } C = c - q_i$$

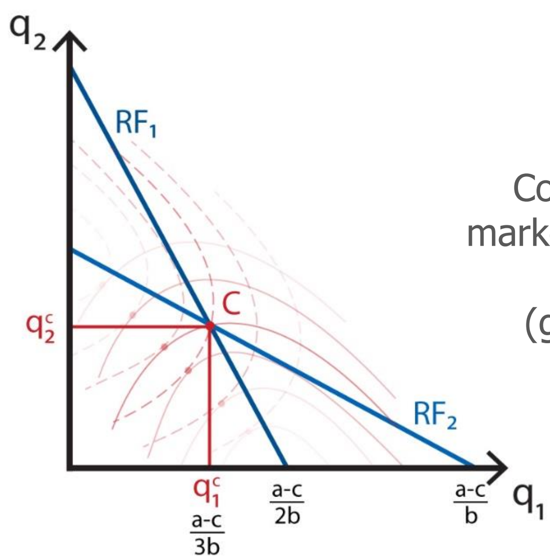
$$\max_{q_i} \pi_i [a - b(q_i + q_j) - c] q_i$$

$$\sigma_i(q_j) = \frac{a - bq_j - c}{2b} = \frac{a - c}{3b}$$

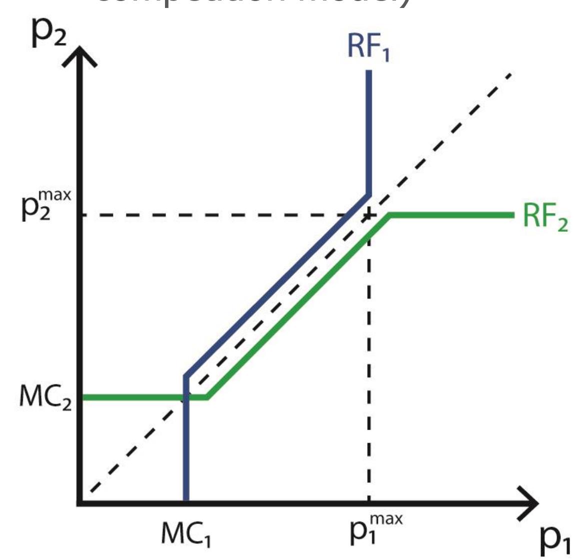
Q(monopoly) < Q(Cournot) < Q(PC)

$$q_1 = D(p_1, p_2) = \begin{cases} D(p_1)/2 \\ D(p_1) \\ 0 \end{cases}$$

Price = Marginal Cost (replicates perfect competition model)



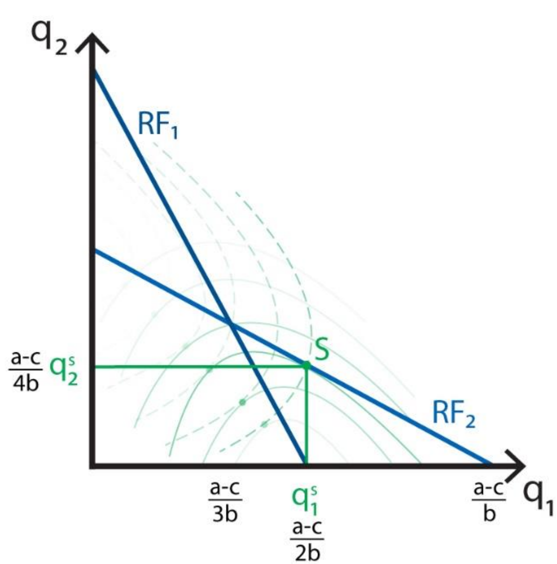
Cournot describes some markets better and vice versa (e.g. oil market is (generalized) Cournot)



Stackelberg duopoly model

Sequential game gives advantage to first player

- Sequential game
- Homogeneous product
- Same demand and cost functions
- Firm 1 moves first and firm 2 after seeing Firm 1's choice reacts
- Solution through backward induction (i.e. we know that the follower has a reaction function as described in the Cournot model; then as the lead firm knows this it will optimize based on the given choice of Firm 2)



$$\sigma_2(q_1) = \frac{a - bq_j - c}{2b}$$

$$\max_{q_1} \pi_1 [a - b(q_1 + \sigma_2(q_1)) - c] q_1$$

$$q_1^* = \frac{a-c}{2b} \qquad q_2^* = \frac{a-c}{4b}$$

Q(SB-2nd) < Q(Cournot) < Q(SB-1st)

Prisoner's dilemma

A prototypical example of modern game theory interaction

	RAT	MUM
RAT	(12, 12)	(1, 24)
MUM	(24, 1)	(6, 6)

Many situations generate the Prisoner's Dilemma outcome. This happens when both players have a dominant strategy that leads to a globally sub-optimal outcome when they independently choose a strategy. Knowing this, players may create institutions to ensure that the globally optimum outcome is imposed

Examples: WTO, Mafia, etc.

$$U_i[(v_i, v_{-i})] = -v_i$$

$$U_1 = [(\mathbf{24}, 1)] < U_1 = [(\mathbf{12}, 12)] < U_1 = [(\mathbf{6}, 6)]$$

$$U_2 = [(1, \mathbf{24})] < U_1 = [(12, \mathbf{12})] < U_1 = [(6, \mathbf{6})]$$

To rat is a dominant strategy:
(1) If the other guy rats, choose 12 or 24 months
(2) If the other guy is mum, choose between 1 or 6 months

In either case, given other player's strategy to rat always dominates mum

Brander-Spencer model

Model of international trade (a case for government subsidies)

	ENTER	OUT
ENTER	(-10, -10)	(50, 0)*
OUT	(0, 50)*	(0, 0)

Domestic subsidy of 20

	ENTER	OUT
ENTER	(10, -10)	(70, 0)*
OUT	(0, 50)	(0, 0)

A case for government subsidizing its domestic firm in the face of international competition – it improves national welfare…

However, if both the domestic and the foreign governments engage in subsidizing their firms then both countries are worse off (though not the firms!)

Brander, James & Barbara Spencer (1981): "Tariffs and the extraction of foreign monopoly rent under potential entry." *The Canadian Journal of Economics*, Vol. 14, Issue 3: 371-89.

Nash equilibrium

John Nash established the existence of equilibria in finite games

THEO. 1: Every finite game has an equilibrium point.

Proof: Using our standard notation, let $\mathcal{s}$ be an n-tuple of mixed strategies, and $p_{i\alpha}(\mathcal{s})$ the pay-off to player i if he uses his pure strategy $\pi_{i\alpha}$ and the others use their respective mixed strategies in $\mathcal{s}$. For each integer λ we define the following continuous functions of $\mathcal{s}$:

$$q_i(\mathcal{s}) = \max_\alpha p_{i\alpha}(\mathcal{s}),$$

$$\phi_{i\alpha}(\mathcal{s},\lambda) = p_{i\alpha}(\mathcal{s}) - q_i(\mathcal{s}) + 1/\lambda, \text{ and}$$

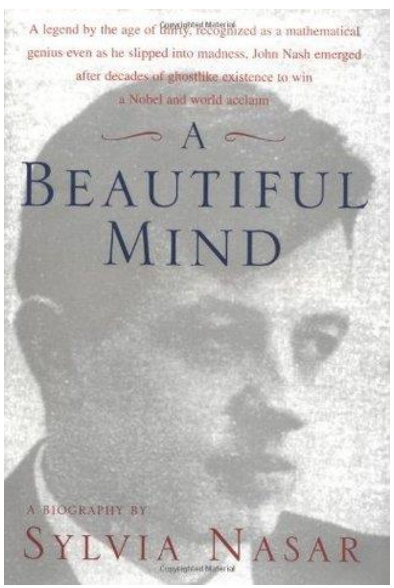
$$\phi^+_{i\alpha}(\mathcal{s},\lambda) = \max\left[0, \phi_{i\alpha}(\mathcal{s},\lambda)\right].$$

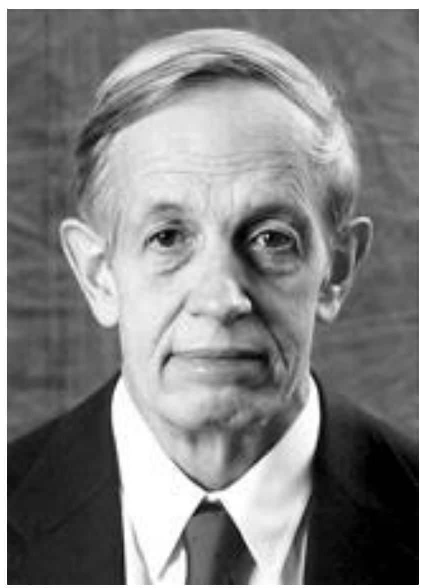
Now $\sum_\alpha \phi^+_{i\alpha}(\mathcal{s},\lambda) \ge \max_\alpha \phi^+_{i\alpha}(\mathcal{s},\lambda) = 1/\lambda > 0$ so that

$$c'_{i\alpha}(\mathcal{s},\lambda) = \frac{\phi^+_{i\alpha}(\mathcal{s},\lambda)}{\sum_\beta \phi^+_{i\beta}(\mathcal{s},\lambda)}$$

is continuous.

A stable state of a non-cooperative game in which no participant can gain by unilaterally deviating from her strategy given the strategies of the other players

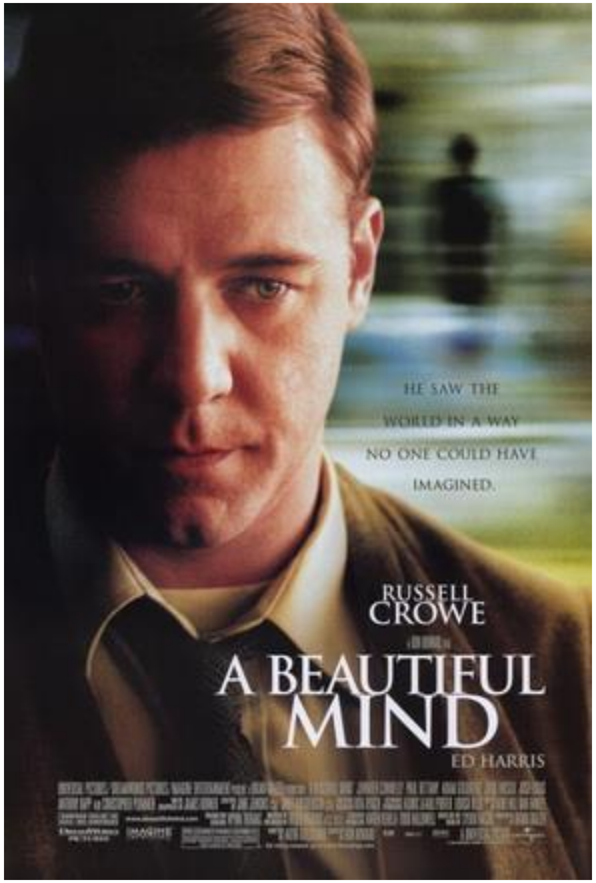


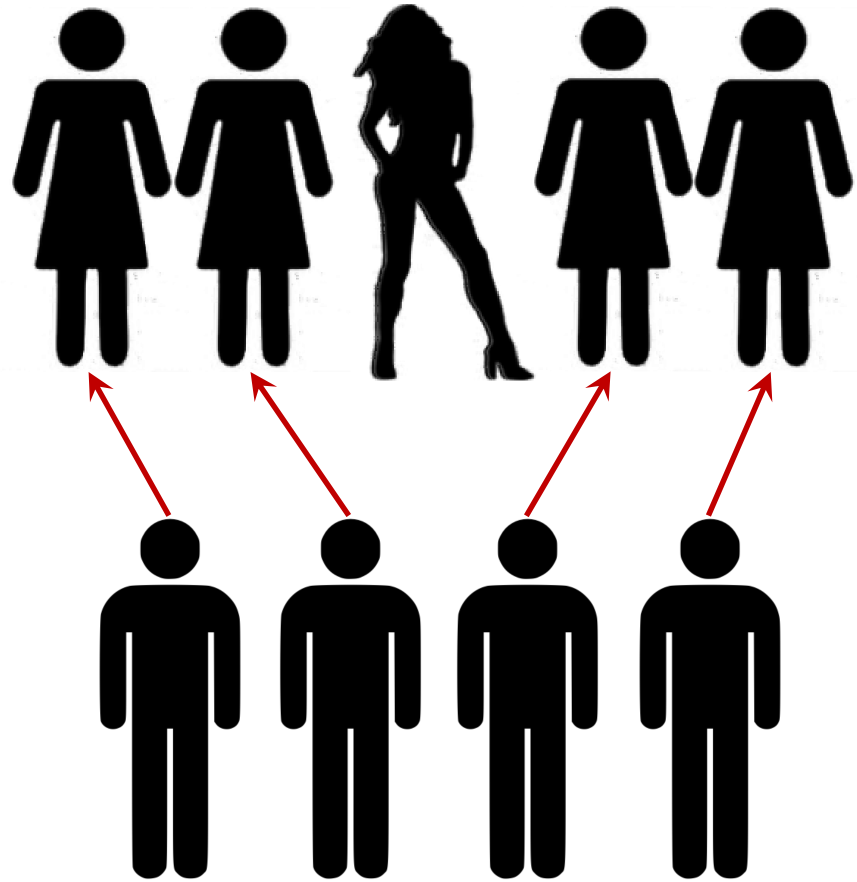


- Nash proved that in any finite non-cooperative game with finite payoffs that there exists an equilibrium
- Pure strategy:
  - Determinant actions
- Mixed strategy:
  - Players choose (a priori) a probability distribution over which to play a pure strategy

Game theory in popular culture

“A Beautiful Mind” (novel and movie)





Prove that the Nash equilibrium, as described (or alluded to) in the move “A Beautiful Mind”, is ***not*** a Nash equilibrium

➔ Moral of the story: Popular culture to is a horrible window for knowledge

Auctions

Not all auctions yield the same results!

SELECTED AUCTION TYPES

Metric	English	Dutch	First-price	Second-price
Winner	Last bidder	First bidder	Highest bidder	Highest bidder
Winner pays	Highest bid	First bid	Highest bid (own)	Second highest bid
Dominant strategy	$\min\{v_i, \max\{v_{-i}\}\}$	Anyone know?	None	Reveal true value
Bidding structure	Ascending price	Descending price	Sealed bid	Sealed bid

Winner will overpay for good when there is incomplete information:
(1) Winning bid exceeds the true value (absolute loss)
(2) Value of good lower than expected but still exceeds bid price (relative loss)

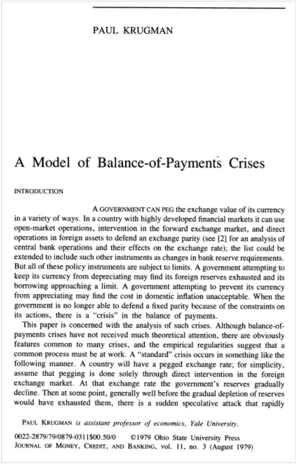
Application 1: Macroeconomics

Speculative currency attacks / currency crises

	RUN	LONG
RUN	(5, 5)	(10, 0)
LONG	(0, 10)	(20, 20)

Although not formally derived from game theory, attacks on currencies have an element of game theory (Paul Krugman, 1979)

- 2nd1st generation: abandon peg given non-sustainable policies
- generation: self-fulfilling prophecies
- 3rd generation: common knowledge



PAUL KRUGMAN

A Model of Balance-of-Payments Crises

INTRODUCTION

A GOVERNMENT CAN PEG the exchange value of its currency in a variety of ways. In a country with highly developed financial markets it can use open-market operations, intervention in the forward exchange market, and direct operations in foreign assets to defend an exchange parity (see [2] for an analysis of central bank operations and their effects on the exchange rate); the list could be extended to include such other instruments as changes in bank reserve requirements. But all of these policy instruments are subject to limits. A government attempting to keep its currency from depreciating may find its foreign reserves exhausted and its borrowing approaching a limit. A government attempting to prevent its currency from appreciating may find the cost in domestic inflation unacceptable. When the government is no longer able to defend a fixed parity because of the constraints on its actions, there is a "crisis" in the balance of payments.

This paper is concerned with the analysis of such crises. Although balance-of-payments crises have not received much theoretical attention, there are obviously features common to many crises, and the empirical regularities suggest that a common process must be at work. A "standard" crisis occurs in something like the following manner. A country will have a pegged exchange rate; for simplicity, assume that pegging is done solely through direct intervention in the foreign exchange market. At that exchange rate the government's reserves gradually decline. Then at some point, generally well before the gradual depletion of reserves would have exhausted them, there is a sudden speculative attack that rapidly

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JOURNAL OF MONEY, CREDIT, AND BANKING, vol. 11, no. 3 (August 1979)

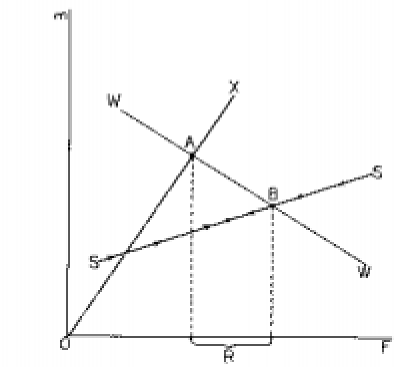
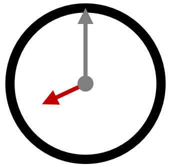
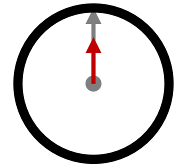
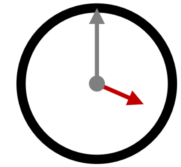


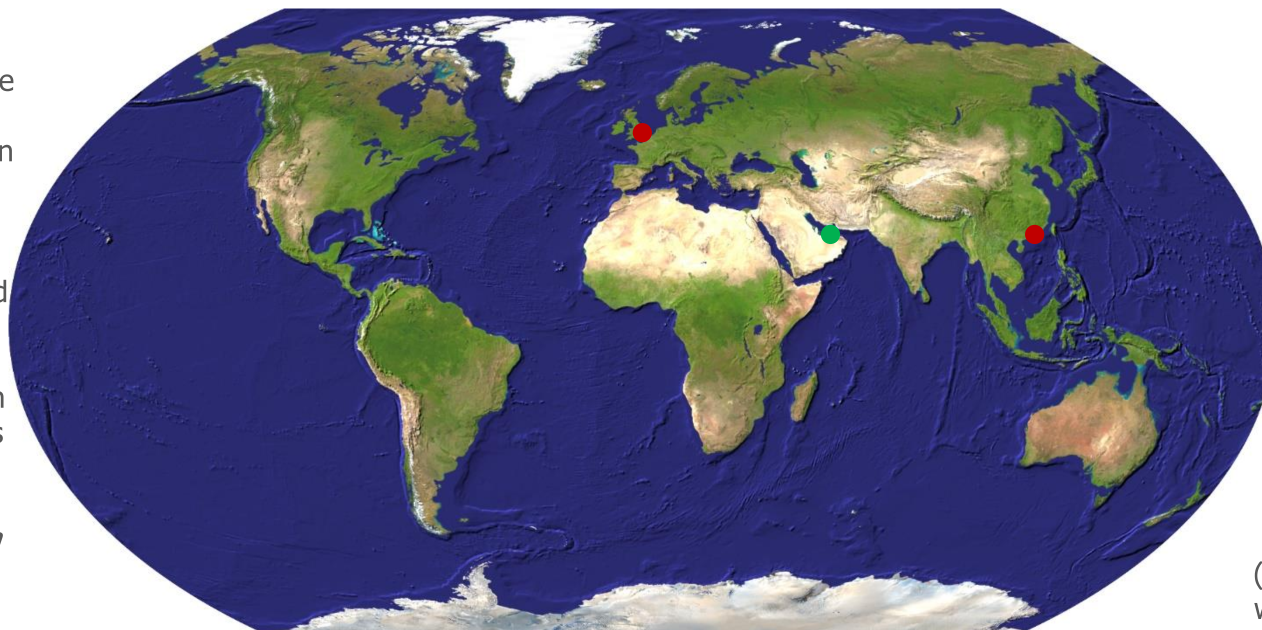
Fig. 5. The Elimination of Reserves by a Speculative Attack

Application 2: Policy

Choosing industrial policy or strategic development plan

Strategically occupying the middle time zone between London and Hong Kong, Dubai lies at the crossroad of Europe, Asia, and Africa. British influence has made **English** the *lingua franca* in the country.



One third of the world's population within a 4-hour flight of Dubai; two-thirds within an 8-hour flight. A natural gateway to Middle East, North Africa and South Asia (MENASA), as well as Africa.

Application 3: Business (technology)

The innovations adapted by the market are not always optimal

In the 1970s two companies vied for the videocassette market based on different technologies:

1. Sony: VHS
2. JVC: Betamax

What is the Nash Equilibrium of the game?

	VHS	***Betamax***
VHS	**(2, 2)**	**(-1, -1)**
Betamax	**(-1, -1)**	**(2, 2)**

- Sony won the war and many movie aficionados were left with a machine that few video producers brought to market
- The market does not always (or more properly is seldom the case?) achieve the socially optimal outcome

Application 4: Behavioural economics

Ultimatum game shows that human behavior is not always rational

Ultimatum game

- 2 players split \$100
- Player 1 chooses $s_1 \in [0{,}100]$ leaving Player 2 with $s_2 = 100 - s_1$
- If Player accepts s_2 then both keep the amounts; otherwise both get zero

Splitting the dollar game:

- 2 players split \$100
- Each player (simultaneously) chooses a number $s_i \in [0{,}100]; i = 1{,}2$
- Each gets the value they choose (under certain conditions – see below)
- Assume utility payoff is equal to value chosen
- Game 1: if $s_1 + s_2 > 100$ then both receive zero
- Game 2: if $s_1 + s_2 > 100$ then winner is $\min\{s_i\}$ and keeps her chosen amount; if $s_1 + s_2 > 100$ and $s_1 = s_2$ then each gets \$50
- Now assume that $s_i \in \{1{,}2{,}3, \dots, 99{,}100\}$
- Can you solve for the Nash equilibria? (How do we know the NE exists?)

Application 5: Politics

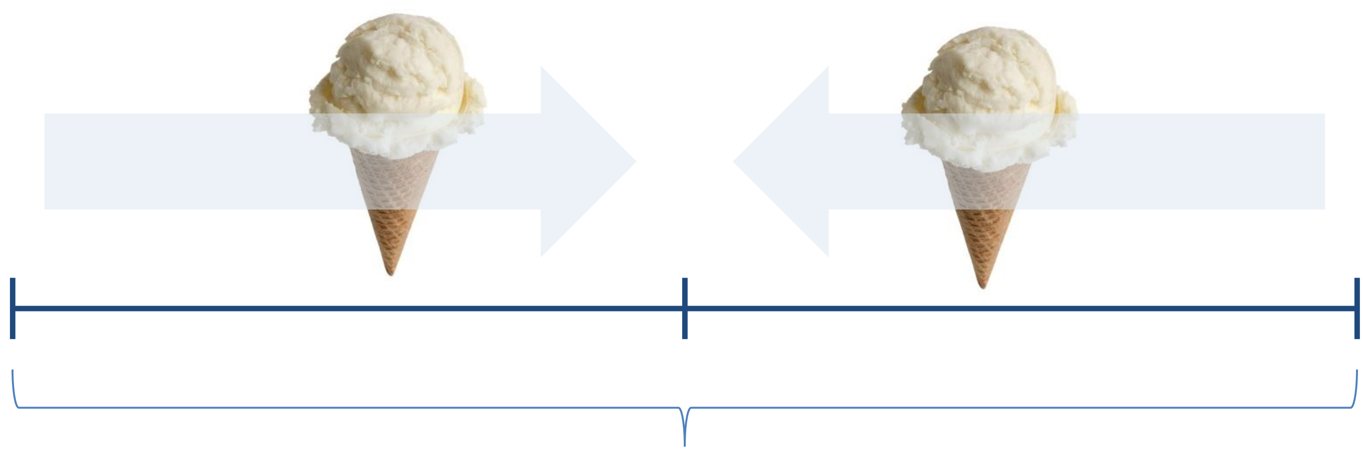
Median voter theorem (derived from ice cream vendors on a boardwalk)

Where an outcome is decide by majority rule, the chosen action is the one most preferred by the median voter (*c.f.* Hotelling, 1929)

Some assumptions of the median voter theorem:

1. Outcomes can be mapped into a one-dimensional space (left vs right / liberal vs conservative)
2. Voters preferences are single-peaked and so choose option closes to them
3. Outcome is decided by majority rule and everyone votes / participates

But then how to explain the growing partisanship in politics?



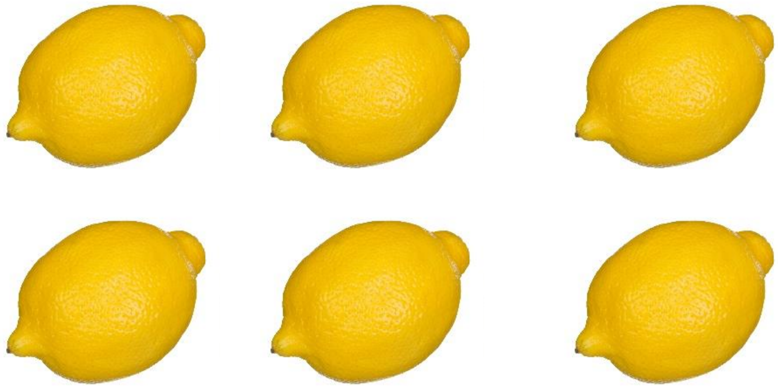
- Q: Is there a steady state for n ≥ 3?

Application 6: Finance (fund raising)

Asymmetric information about value of company

In finance, raising money for a start-up company has many intricacies / details:

- Firm needs cash to grow, but need to ask why it needs cash
  - If because of weak finances then creates adverse selection in market
  - If because of limited capacity to scale up profitable business then positive selection
- Firm wants to raise money with highest valuation possible while giving up least control of company
- Investor wants to acquire largest share possible but at lowest price
- What signal does price give?
- How to model this?



Suggested reading:

Diamond-Dybvig model (1983)

Diamond and Dybvig (JPE, 1983)

Assumptions

- single homogeneous good
- 3 periods. Project is divisible.

T=0	T=1	T=2
-1	0	R
	1	0
investment	When production is interrupted (salvage value)	R>1

- 2 types of agents:

Type	1	2
1	short-term	
2		long-term

  - All consumers are identical at T=0.
  - At T=1, they learn about their type (Type 1 or Type 2) which is privately observed.
  - Type 1 cares about consumption in period 1 and Type 2 in period 2.

- State dependent utility, where state θ depends on type.

$$u(c_1, c_2; \theta) = \begin{cases} u(c_1) & \text{if state } \theta \text{ is Type 1} \\ \rho u(c_1 + c_2) & \text{if state } \theta \text{ is Type 2} \end{cases}$$

where ρ is a discount rate and $1 \geq \rho > 1/R$. ($\rho R > 1$)

  - c_T = goods received (to store or consume) by agent at period T.
  - For Type 2, the privately observed consumption at T=2 is $(c_1 + c_2)$, where c_1 is what is stored at T=1 and c_2 is what is obtained at T=2 (from investment).

- Agents:
  - Risk averse agents: $-cu''(c)/u'(c) > 1$
  - Agents maximize $Eu(c_1, c_2; \theta)$.
  - t = fraction of all agents who are Type 1. $t \in (0,1)$. This is also the probability of an agent being Type 1 at T=1.
  - Agents receive endowment of 1 unit at period 0 only.

a) Competitive solution

- Result: agents will not trade and cannot get the first best solution where types are publicly observable.
- Assumptions:
  - Types are not publicly observable → no insurance contract available.
  - In each period, there is a competitive market in claims on future goods.

Agents can issue and trade a non-contingent asset based on the project/production technology. Assets cannot be state contingent because types are not publicly observable. Prices/returns are determined by the project technology.
The period 0 price of consumption at period 1 should be 1, because the return at period 1 on the trade of consumption cannot be greater than the return on the production technology (salvage value) and cannot be smaller than the return on storage. The period 0 and 1 price of consumption at period 2 should be $1/R$ (the return is R), because the return cannot be greater than the return on the production technology and cannot be smaller than the return agents can obtain through their private production.

With the price set of consumption, there is no trade of assets. Agents invest privately and liquidate when Type 1 and continue production when Type 2. Denote consumption of Type *j* at period *T* by C_T^j. Agents choose

$$C_1^1 = 1, C_2^2 = R,$$
$$C_1^2 = C_2^1 = 0.$$

This is the competitive outcome, but not first best as shown below.

b) If types were publicly observable as of period 1

- Assumptions
  - Types are observable → insurance contract available based on types (state contingent Arrow-Debreu securities are traded)

Because markets are complete, we solve the planner's problem instead of the consumer's problem.

$$\max_{C_1^1, C_2^2} \quad tu(C_1^1) + \rho(1-t)u(C_2^2)$$

$$\text{subject to} \quad tC_1^1 + (1-t)\tfrac{1}{R}C_2^2 = 1 \qquad (1)$$

First order conditions are