MPP Calculus Problem Set 5

1. Method of Lagrange multipliers

1.1. Optimisation with equality constraints

Chiang, Chapter 12.

• Exercise 12.2: 1 (a, c), 3 (all).

• Exercise 12.5: 1 (all), 2 (all).

• Solve the following maximisation problems:

1. Maximise $z = xy$ subject to $z = x + y = 10$.
2. Maximise $z = x^2 + y^2$ subject to $x + 2y = 5$.
3. Maximise $U(x_1, x_2) = \alpha \log x_1 + (1 - \alpha) \log x_2$ subject to $p_1 x_1 + p_2 x_2 = M$.

1.2. Optimisation with inequality constraints

Chiang, Chapter 21.

• Exercise 21.1: 1, 2, 3.

• Exercise 21.4: 1 (all), 4 (all).

• Exercise 21.6: 4, (all), 5 (all).

• Solve the following optimisation problems:

1. Maximise $U = U(x_1, \dots, x_n)$ subject to $p_1 x_1 + \cdots + p_n x_n \leq M$ and $x_1, \dots, x_n \geq 0$.
2. Maximise $U(x_1, \dots, x_n) = x_1 x_2 + 2x_1$ subject to $p_1 x_1 + \cdots + p_n x_n \leq M$, and $x_1, \dots, x_n \geq 0$, $n \geq 2$.
3. Minimise $C = rK + wL$ subject to $K^\alpha L^{1-\alpha} > \bar{Q} > 0$, and $K > 0$, $L > \bar{L}$.