MPP Calculus Problem Set 2

Summer 2002

Due: Wednesday, August 21.

1. Factoring

Factor the following quadratics:

1. $x^2 + x - 2$
2. $x^2 + 4x - 21$
3. $x^2 - 16x + 39$
4. $x^2 - 14x - 51$
5. $12x^2 - 23x - 24$ (Hint: combinations whose product is 12 are (1; 12), (2; 6), and (3; 4), and for 24 are (1; 24), (2; 12), (3; 8), and (4; 6))

2. Derivatives

Find the following derivatives from first principles; i.e. by using the fact that: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

1. $f(x) = x^2$
2. $f(x) = 1/x$ (Hint: See Spivak, Chapter 9, question 1(a))
3. $f(x) = x^n$ for some $n \in \boldsymbol{R}$
4. $f(x) = ax^3$ for some constant a
5. $f(x) = ax^{1/2}$ (Hint: See Spivak, Chapter 9, question 3)

Spivak: Chapter 9: 5, 14, 26.

3. Differentiation

Find the first and second derivatives of the following functions. (There is no need to find these derivatives by first principles.)

1. $f(x) = x^3 e^{x^3}$
2. $f(x) = [x^2 + x - 4]/[\log(1 + x^2) - e^x]$
3. $f(x) = x^\alpha y^{1-\alpha}/(1-\alpha)$ where y and α are constants.
4. $f(x) = x\sqrt{x - \exp[(4x-1)/(3x)]}$
5. $f(x) = ax^2 \exp(e^x - 1)$
6. $f(x) = (e^x + 1)/(e^x - 1)$

Spivak: Chapter 11: 1 (odd), 3 (even), 9, 10, 48

4. L'Hôpital's Rule

Using L'Hôpital's Rule find the following limits (if they exist):

1. $\lim_{x\to 1}[x^2+2x+1]/[\log(x^3+2)]$

2. $\lim_{\rho\to 1}[C^{\rho-1}-1]\,/(1-\rho)$ (Often in economics you will see that people will write this (utility) function as $C^{\rho-1}/(1-\rho)$. Why is it okay to write the utility function like this and claim that it has the same limiting value as ρ approaches 1?)

3. $\lim_{x\to 1}[x^2+2x+1]\,/[\log(x^3+2)]$

4. $\lim_{x\to 2}[x^3-8]\,/\left\{\exp\left[-\frac{1}{x-2}\right]\right\}^3$

5. In class we remarked that $\frac{\lim_{x\to 1} e^x}{x^n}=1$. Verify this assertion using L'Hôpital's Rule.

5. Critical points

Find the critical points of the following functions and state whether the critical points correspond to minimal or maximal values of the function (or neither).

1. $f(x)=3x^3-2x^2+x-1$ for x ∈ [-10;10]

2. $f(x)=(x^2+2)/(x^2-1)$

3. $f(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$

4. $f(x)=x^5-x^4+x^3-x^2+x-1$

5. $f(x)=\log(\log(\log(\log x)))$

6. Graphing

Graph the following functions:

1. $f(x)=(4x^3-4x-5)/(x^3-1)$

2. $f(x)=e^{x^2}/(x^2-1)$

3. $f(x)=\log(x^x)\,/\,\exp(x)$

7. Inverse functions

Spivak Chapter 12: 2 (all), 4, 6, 7 (all).

8. Economics

Quite often in economics production functions are assumed to satisfy something known as the "Inada conditions":

$$f(0)=0, f'>0, f''<0, \lim_{x\to 0} f'(x)=\infty, \lim_{x\to\infty} f'(x)=0 \qquad (2)$$

Can you find a function *f* that satisfies these equations?