Introductory Statistics
Winter 2004

PRACTICE MIDTERM EXAM

Q1. Suppose the random variable X has p.d.f. $f(x) = x^2$ for $-\sqrt{3/2} \leq x \leq \sqrt{3/2}$. Find E(X). Hint: Sketch the graph of $f(x)$ and use the fulcrum analogy.

Q2. Suppose that the grades for an examination are (approximately) distributed normal with mean 50 and variance 49. That is, $X \sim N(50,7^2)$.

a. The professor wants to raise the class average to 75 by multiplying all test scores by a factor of 3/2. What happens to the variance under this procedure?
b. What if instead she raised the grades by adding a factor of 25 to each test score? That is, how does the variance change under this procedure?
c. With the unadjusted test scores, what cut offs need she make in the raw scores so that only 10 percent get a grade of A and 30 percent a grade of B?

Q3. Suppose the winter daytime (maximum) temperature in Iqaliut (X) is normally distributed with mean –20 and variance 36, i.e., $X \sim N(-20,6^2)$.

a. On any given day, what is the probability that the daytime (maximum) temperature falls in the range (-23,-19)?
b. Over a nine-day period, what is the probability that the average daytime temperature falls within the range (-23,-19)?

Q4. Consider two mutual funds, "Arc Investments" (X) and "Vinyl Funds" (Y), whose returns are both normally distributed: $X \sim N(5,5^2)$ and $Y \sim N(8,8^2)$.

a. If you placed half your wealth in Arc Investments and the other half in Vinyl Funds, what is the probability that your net investment loses money?
b. Which investment is riskier? Explain.

Q5. Suppose that you get tested for the dinopeptic germ and you discover that you have a positive test result (as did Fred Flintstone in the infamous stay-awake-for-24-hours episode). The dinopeptic germ is a rather rare in humans, occurring in only 1 percent of the population. Your doctor tells you that the test is 95 percent accurate (i.e., gives a positive test 95 percent of time given you truly have the condition) and gives a false positive only 1 percent of the time. What is the probability that you truly have the dinopeptic given your positive test (and hence the need to stay awake 24 hours)?

Q6. You roll a die ten times.

a. How many different ways are there of achieving four 6s in these ten rolls?
b. What is the probability of getting at least one hundred 6s in 590 rolls?

Q7. Consider the following probability table below:

	$X = 0$	$X = 1$
$Y = 0$	0.20	0.20
$Y = 1$	0.10	0.25
$Y = 2$	0.15	0.10

(a) Find the expected value of X: $E(X)$.
(b) Find the expected value of X given that $Y = 1$: $E(X|Y=1)$.
(c) Find the variance of X: $Var(X)$.

Q8. Many international relief agencies take a-dollar-a-day as the measure for counting those living in poverty. What type of data is the poverty headcount? And why is it problematic as a way of targeting the poor?

Q9. You are presented with the three following scatter diagrams. Take it for granted that there are no distortions in the graphs.

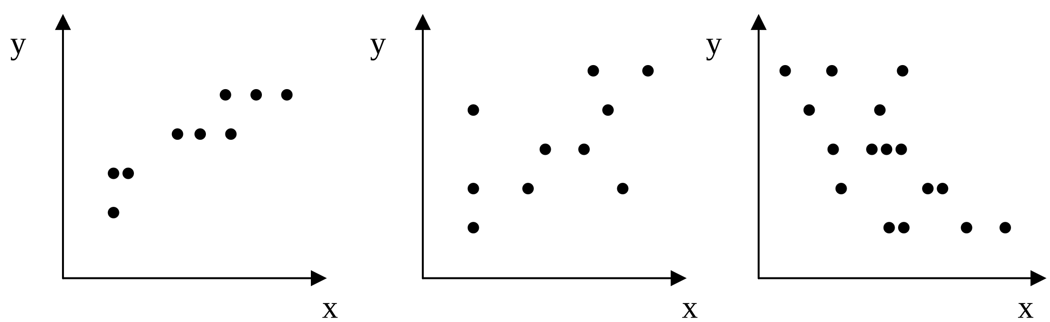

Chart 1 Chart 2 Chart 3

a. Which of these charts has the highest absolute correlation?
b. Which of these charts shows a negative correlation?
c. If the scatter diagram is non-linear should we abandon linear regression analysis?

Q10. Suppose that the lifetime of a light bulb (in hours) has exponential distribution

$$f(x) = (\lambda)\exp(-x\lambda)$$

where $\lambda = 1/3000$. That is

$$f(x) = (1/3000)\exp(-x/3000)$$

What is the expected value of X (i.e., the average lifetime of the light bulb)?

Q11. Suppose that the arrival time of a bus (in minutes) at a bus stop is distributed with a p.d.f. $f(x) = (15-0.5x)/225$ for $0 \leq x \leq 30$. You arrive at the bus stop and the bus just left.

a. What is the probability that the bus will come in the next five minutes?
b. After 5 minutes the bus still hasn't come. What is the probability that the bus will come in the next five minutes (given that you have already waited five minutes)?

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Bonus: Recall the game in which Betty and Wilma play a rather unusual game (tossing a coin until the first H appears) that has an expected payoff of infinite. What if we instead played the same game but with a smaller payoff? In particular, suppose the payoffs are such that: they win $2 if it come up H on the first toss; $2 if they win on the second toss; $(8/3) if they win on the third toss; $(16/4) if they win on the fourth toss; $(32/5) if they win on the fifth toss; etc. What is the expected value of this gamble?