Introductory Statistics
Winter 2004

MIDTERM EXAM
February 24, 2004

You have 75 minutes to answer the following questions. Each of the first five questions is worth 15 points. The five remaining questions are worth 5 points apiece. There is one bonus question at the end worth 10 points. The test is thus out of 100 with 10 possible bonus points. Allocate your time accordingly. <u>*You must show your work!*</u>

15 points
Q1. The makers of "Weight Gain 4000" have decided to market their product through the medium of infomercials. A call centre is set up to handle incoming calls from wannabe "beefcakes". Suppose that the number of phone calls coming in to the call centre (per hour) is described by a Poisson distribution with parameter $\mu = 16$. That is

$$p(x) = \frac{e^{-16}16^x}{x!} \quad x = 0,1,2,3,\ldots$$

(a) What is the probability that the call centre will receive exactly one phone call in the next hour?
(b) What is the probability that the call centre will receive at least four calls in the next hour?
(c) Using Chebyshev's theorem, find an upper bound for the probability that the call centre will receive 28 or more calls in the next hour. Hint: If X is distributed Poisson with parameter μ, then $\text{Var}(X) = \mu$.

15 points
Q2. Consider two mutual funds, "Howells Investments" (X) and "Riveras Funds" (Y), whose returns are both normally distributed: $X \sim N(10,5^2)$ and $Y \sim N(15,8^2)$.

(a) What is the probability that Howells Investment earns at least 15 percent? Likewise, what is the probability that Riveras Funds earns at least 15 percent?
(b) If an investor is very nervous about earning a negative return and has to choose between the two funds, which one should she pick?
(c) If an investor placed 1/3 of her money in Howells Investments and the remainder of her money in Riveras Funds, what is the expected return on her investment?

15 points

Q3. Consider the following probability table below:

	$X = 0$	$X = 1$
$Y = 0$	0.20	0.30
$Y = 1$	0.10	0.40

(a) Find the expected value of X: E(X).
(b) Find the expected value of X given that $Y = 1$: E($X|Y$=1).
(c) Find the variance of X: Var(X).

15 points

Q4. Fred and Barney are in Rock Vegas living life in the fast lane. They are at a casino table where the object of the game is to roll a die. A winning roll is one in which one dot appears. Suppose that the die is fair.

(a) What is the probability that it takes 6 rolls before they get their first winning roll?
(b) What is the probability that in 10 rolls that they get 3 winning rolls?
(c) If the house pays out $6 for a winning roll, what ante must Fred and Barney put up to make it a fair game?

15 points

Q5. Adult North American male weight is (approximately) distributed normal with mean 80 kilograms and variance 100 kilograms^2. That is, W ~ N(80,10^2). An elevator has a capacity of 725 kg before the cable snaps.

(a) If 10 random people get on the elevator, what is the probability that they will "go for a ride"? (Hint: What must be true of the sample mean weight for the cable to snap?)
(b) What does the central limit theorem (CLT) say about the distribution of the sample mean of weight, even if we know nothing about the distribution of weight?
(c) If the population parameters of weight are unknown, and thus estimated with sample statistics, the underlying statistics have a student-t distribution. What happens to the t distribution as the sample size increases?

5 points.

Q6. Suppose that the arrival time of a bus at a bus stop is uniformly distributed over a 30 minute range: X ~ U[0,30]. You arrive at the bus stop. What is the probability that a bus will come in the next 5 minutes?

5 points

Q7. The article “Filling the World’s Belly” (The Economist, 2003) discussed the growing problem of obesity. Obesity is defined through a body mass index (BMI) that takes a person’s weight in kilograms, divided by height, in metres, squared. That is, BMI = kg/m^2. A person is said to be obese if their BMI is over 30. In the United States, 32 percent of adults are considered obese. In Canada the percent of obese adults is 12 percent. Knowing this is it fair to say that there are many more fat people in Canada than in the United States? Explain.

5 points

Q8. In the article “Where Has All the Money Gone?” (The Milken Institute) the author argues that the average American is not much richer today than in the beginning of the 90s in spite of the fact that mean income has increased significantly. What was his argument in making this assertion?

5 points

Q9. The article “The War of the Headscarves” (The Economist, 2004) discussed the issue of headscarves and other conspicuous forms of religious attire being banned. A poll conducted by CSA/Le Parisien asked a sample of people, “Are you in favour of, or opposed to, a law banning signs or dress that conspicuously display religious affiliation?” The result is listed below:

	In favour, %	Opposed, %
All French	69	29
Left	66	33
Right	75	24
Muslims	42	53
Muslim women	49	43

What percent of Muslim men are opposed to the new law? (You may assume that an equal number of male and female Muslims were surveyed.)

5 points

Q10. Use a Venn diagram to argue that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

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10 points

Bonus: Consider two mutual funds, “Wall Street Boyz” and “Lombardi Street Girlz”. You have W dollars to invest in these two funds. Your wealth is described by the function $W = \alpha X + (1-\alpha)Y$. Where αX is share of wealth in “Wall Street Boyz” and $(1-\alpha)Y$ is share of wealth placed in “Lombardi Street Girlz”, and $\alpha \in [0,1]$. You are a very conservative investor, and your only objective is to minimise the variance of your portfolio. Suppose that:

(i) $\text{Var}(X) = 1$; (ii) $\text{Var}(Y) = 2$; (iii) $\text{Cov}(X,Y) = -1$.

Find the value of α that minimises the variance of your portfolio.

Introductory Statistics

MIDTERM EXAM
March 2, 2004

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<u>When you are done you must place this test paper in your exam booklet!</u>

Q1. (15 points) Consider two mutual funds, "Sound Factory Mutual" (X) and "Vinyl Funds" (Y), whose returns are both normally distributed: $X \sim N(9,5^2)$ and $Y \sim N(12,8^2)$.

(d) What is the probability that Sound Factory Mutual earns at least 10 percent? Likewise, what is the probability that Vinyl Funds earns at least 10 percent?
(e) What is the probability that Sound Factory Mutual will earn between 0 and 9 percent?
(f) If an investor placed 1/4 of her money in Sound Factory Mutual and the remainder of her money in Vinyl Funds, what is the expected return on her investment?

Q2. (15 points) Over the course of 30 days the makers of Weight Gain 4000 ("Je veux être un gâteau à la viande!") have received 480 calls from interested body builders. Let X be the number of calls per day to the call centre handling the orders for WG4000, and suppose that X follows a Poisson distribution.

$$p(x) = \frac{e^{-\mu}\mu^{x}}{x!} \quad x = 0,1,2,3,\ldots$$

(d) Find the value of μ for this question.
(e) What is the probability that the call centre will receive at least four calls in a given day? (You need not simplify your answer.)
(f) Using Chebyshev's theorem, find an upper bound for the probability that the call centre will receive 24 or more calls in the next hour. (Hint: If X is distributed Poisson with parameter μ, then $\mathrm{Var}(X) = \mu$.)

Q3. (15 points) Consider the following probability table below:

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	0.20	0.15	0.10
$Y = 1$	0.10	0.20	0.25

(d) Find the expected value of X: $E(X)$.
(e) Find the expected value of X given that $Y = 1$: $E(X|Y=1)$.
(f) Find $E(X^2)$.

Q4. (15 points) Suppose that the you have a probability density function given by $f(x) = |x|$ over the interval [-1,1], where $|x|$ is the absolute value of x: $|x| = \{x \text{ if } x \geq 0; -x \text{ if } x < 0\}$

a. Find $E(X)$.
b. Verify that $f(x) = |x|$, for $-1 \leq x \leq 1$, is indeed a proper pdf (two conditions).
c. Find $P(-0.5 < X \leq 0.5)$

Q5. (15 points) Suppose that the grades for an examination are (approximately) distributed normal with mean 60 and variance 49. That is, $X \sim N(60,7^2)$.

a. The professor wants to raise the class average to 80 by multiplying all test scores by a factor of 4/3. What happens to the variance under this procedure?
b. What if instead she raised the grades by adding a factor of 20 to each test score? That is, how does the variance change under this procedure?
c. With the unadjusted test scores, what cut offs need she make if she wishes to flunk (F) 10 percent of the class?

Q6. (5 points) Suppose that the arrival time of a bus at a bus stop is uniformly distributed over a 15 minute range: $X \sim U[0,15]$. You arrive at the bus stop. What is the probability that a bus will come in the next 15 minutes?

Q7. (5 points) The article "Filling the World's Belly" (The Economist, 2003) discussed the growing problem of obesity. In the United States, 32 percent of adults are considered obese. In Canada the percent of obese adults is 12 percent. There are about 300 million Americans and 30 million Canadians in North America. You meet an obese North American on an airplane. What is the probability that this person is American?

Q8. (5 points) The article "The War of the Headscarves" (The Economist, 2004) discussed the issue of headscarves and other conspicuous forms of religious attire being banned. A poll conducted by *CSA/Le Parisien* asked a sample of people, "Are you in favour of, or opposed to, a law banning signs or dress that conspicuously display religious affiliation?" The result is listed below:

	In favour, %	Opposed, %
All French	69	29
Left	66	33
Right	75	24
Muslims	42	53
Muslim women	49	43

What percent of Muslim men are in favour of the new law? (Assume that an equal number of male and female Muslims were surveyed.)

Q9. (5 points) State the central limit theorem.

Q10. (5 points) In the article "Where Has All the Money Gone?" (The Milken Institute), the author argues that the average American is not much richer today than in the beginning of the 90s in spite of the fact that mean income has increased significantly. What was his argument in making this assertion?

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Bonus. (10 points) Consider two mutual funds, "Wall Street Boyz" and "Lombardi Street Girlz". You have W dollars to invest in these two funds. Your wealth is described by the function $W = \alpha X + (1-\alpha)Y$. Where αX is share of wealth in "Wall Street Boyz" and $(1-\alpha)Y$ is share of wealth placed in "Lombardi Street Girlz", and $\alpha \in [0,1]$. You are a very conservative investor, and your only objective is to minimise the variance of your portfolio. Suppose that:

(i) $\text{Var}(X) = 1$; (ii) $\text{Var}(Y) = 2$; (iii) $\text{Cov}(X,Y) = -1$.

Find the value of α that minimises the variance of your portfolio.

<u>*Super Bonus*</u>. (5 points) Hannah Barbara Productions originally had intended that Fred and Wilma, of the "Flintstones" cartoon, were to have a baby boy. As we all know, in the end they decided on a having a female baby, "Pebbles" (and in my opinion it was the best damn decision they ever made). What name was proposed for the male baby that never came to fruition?

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<u>When you are done you must place this test paper in your exam booklet!</u>

Q1. (15 points) Fred and Barney are in Rock Vegas living life in the fast lane. They are at a casino table where the object of the game is to roll a die. A winning roll is one in which one dot appears (i.e., "1"). Suppose that the die is fair.

(d) What is the probability that it takes 6 rolls before they get their first winning roll? (Hint: What must be true of the first five rolls, and then that of the sixth?)
(e) What is the probability that in 10 rolls that they get exactly 3 winning rolls? (You need not simplify your answer.)
(f) If the house pays out $6 for a winning roll, what ante must Fred and Barney put down to make it a fair game?

Q2. (15 points) Consider two mutual funds, "Howells Investments" (X) and "Riveras Funds" (Y), whose returns are both normally distributed: $X \sim N(10,5^2)$ and $Y \sim N(15,8^2)$.

(g) What is the probability that Howells Investment earns at least 15 percent? Likewise, what is the probability that Riveras Funds earns at least 15 percent?
(h) If an investor is very nervous about earning a negative return and has to choose between the two funds, which one should she pick?
(i) If an investor placed 1/3 of her money in Howells Investments and the remainder of her money in Riveras Funds, what is the expected return on her investment?

Q3. (15 points) Consider the following probability table below:

	$X = 0$	$X = 1$
$Y = 0$	0.20	0.30
$Y = 1$	0.10	0.40

(g) Find the expected value of X: $E(X)$.
(h) Find the expected value of X given that $Y = 1$: $E(X|Y=1)$.
(i) Find the variance of X: $\mathrm{Var}(X)$.

Q4. (15 points) The makers of “Weight Gain 4000” have decided to market their product through the medium of infomercials. A call centre is set up to handle incoming calls from wannabe “beefcakes”. Suppose that the number of phone calls coming in to the call centre (per hour) is described by a Poisson distribution with parameter $\mu = 16$. That is

$$p(x) = \frac{e^{-16}16^{x}}{x!} \quad x = 0,1,2,3,\ldots$$

(g) What is the probability that the call centre will receive exactly one phone call in the next hour? (You need not simplify your answer.)
(h) What is the probability that the call centre will receive at least four calls in the next hour? (You need not simplify your answer.)
(i) Using Chebyshev’s theorem, find an upper bound for the probability that the call centre will receive 28 or more calls in the next hour. (Hint: If X is distributed Poisson with parameter μ, then $\text{Var}(X) = \mu$.)

Q5. (15 points) Adult North American male weight is (approximately) distributed normal with mean 80 kilograms and variance 100 kilograms^2. That is, $W \sim N(80,10^2)$. An elevator has a capacity of 729 kg before the cable snaps.

(d) If 9 random people get on the elevator, what is the probability that they will “go for a ride”? (Hint: What must be true of the sample mean of weight for the cable to snap?)
(e) What does the central limit theorem (CLT) say about the distribution of the sample mean of weight, even if we know nothing about the distribution of weight?
(f) If the population parameters of weight are unknown, and thus estimated with sample statistics, the underlying statistics have a student *t*-distribution. What happens to the *t*-distribution as the sample size increases?

Q6. (5 points) Suppose that the arrival time of a bus at a bus stop is uniformly distributed over a 30 minute range: $X \sim U[0,30]$. You arrive at the bus stop. What is the probability that a bus will come in the next 5 minutes?

Q7. (5 points) The article “Filling the World’s Belly” (The Economist, 2003) discussed the growing problem of obesity. Obesity is defined through a body mass index (BMI) that takes a person’s weight in kilograms, and divides it by height, in metres, squared; i.e., BMI = kg/m^2. (For example, a person who is 172cm tall and weighs 70kg has a BMI of $70/(1.72)^2 = 23.7$.) A person is said to be obese if their BMI is over 30. In the United States, 32 percent of adults are considered obese. In Canada the percent of obese adults is 12 percent. Knowing this is it fair to say that the United States has a far more severe weight problem? Explain.

Q8. (5 points) In the article "Where Has All the Money Gone?" (The Milken Institute), the author argues that the average American is not much richer today than in the beginning of the 90s in spite of the fact that mean income has increased significantly. What was his argument in making this assertion?

Q9. (5 points) The article "The War of the Headscarves" (The Economist, 2004) discussed the issue of headscarves and other conspicuous forms of religious attire being banned in public schools in France. A poll conducted by *CSA/Le Parisien* asked a sample of people, "Are you in favour of, or opposed to, a law banning signs or dress that conspicuously display religious affiliation?" The result is listed below:

	In favour, %	Opposed, %
All French	69	29
Left	66	33
Right	75	24
Muslims	42	53
Muslim women	49	43

What percent of Muslim men are opposed to the new law? (Assume that an equal number of male and female Muslims were surveyed.)

Q10. (5 points) Use a Venn diagram to argue that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

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Bonus. (10 points) Consider two mutual funds, "Wall Street Boyz" and "Lombardi Street Girlz". You have W dollars to invest in these two funds. Your wealth is described by the function $W = \alpha X + (1-\alpha)Y$. Where αX is share of wealth in "Wall Street Boyz" and $(1-\alpha)Y$ is share of wealth placed in "Lombardi Street Girlz", and $\alpha \in [0,1]$. You are a very conservative investor, and your only objective is to minimise the variance of your portfolio. Suppose that:

(i) $Var(X) = 1$; (ii) $Var(Y) = 2$; (iii) $Cov(X,Y) = -1$.

Find the value of α that minimises the variance of your portfolio.

Super Bonus. (5 points) In the movie "A Beautiful Mind", starring Russell Crowe as the eccentric Princeton mathematician John Nash, there was a scene in the movie in which Crowe's character tries to explain to his friends the idea of game theory. In particular, he described a strategy in which he and his graduate student colleagues would be able to find themselves dates in the scenario when a beautiful girl along with some of her so-so friends walks into a bar. Why were most economists furious (or at least exasperated) at the analogy used in the movie to describe a "Nash equilibrium"?

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Q1. (15 points) The makers of "Ginsu 2000" (kitchen knives) have decided to market their product through the medium of infomercials. A call centre is set up to handle incoming calls from wannabe sashimi samurais. Suppose that the number of phone calls coming in to the call centre (per hour) is described by a Poisson distribution with parameter $\mu = 9$. That is

$$p(x) = \frac{e^{-9}9^x}{x!} \quad x = 0,1,2,3,\ldots$$

(j) What is the probability that the call centre will receive at most one phone call in the next hour? (You need not simplify your answer.)
(k) What is the probability that the call centre will receive at least four calls in the next hour? (You need not simplify your answer.)
(l) Using Chebyshev's theorem, find an upper bound for the probability that the call centre will receive 15 or more calls in the next hour. (Hint: If X is distributed Poisson with parameter μ, then $\text{Var}(X) = \mu$.)

Q2. (15 points) Fred and Barney are in Rock Vegas living life in the fast lane. They are at a casino table playing a card game. They are dealt five cards from the deck (without replacement). There are thirteen cards of each of four suits in a deck of cards (ignore jokers, and thus a total of 52 cards), and assume that the cards are sufficiently shuffled.

(g) What is the probability that they get five cards that are all hearts? (Hint: Use a tree or conditional probability statement.)
(h) Given that the first three cards they are dealt are aces, what is the probability that they will end up with a four-of-a-kind of aces? (Hint: There are two remaining chances that he gets the final ace.)
(i) Down $100 at the end of the night, Fred and Barney try to get out of their debt by playing "double or nothing" on a draw of a card from the (full) deck. If the card suit is spades they owe nothing. Otherwise they owe $200. What is the size of their expected debt from this gamble?

Q3. (15 points) The finishing time of the Toronto Marathon (X) is distributed (approximately) normal with mean 275 minutes and standard deviation 60 minutes: $X \sim N(275, 60^2)$.

(a) Find the probability that a runner finishes the race in exactly 3 hours.
(b) Find the probability that a runner completes the race in less than 4 hours.
(c) You survey 49 random runners. What is the probability that the average finishing time of this group falls in the range of 270 to 280 minutes?

Q4. (15 points) In the kingdom of Kainada live 5 people – call them "*A*", "*B*", "*C*", "*D*", and "*E*". The incomes of *A*, *B*, *C*, *D*, and *E* are \$5, \$10, \$15, \$20, and \$25, respectively.

(a) Find the median and mean incomes in Kainada.
(b) Find the standard deviation of income in Kainada.
(c) Find the median and mean incomes if *E*'s income increases to \$100.

Q5. (15 points) Consider the following probability table below:

	$X = 0$	$X = 1$
$Y = 0$	0.30	0.10
$Y = 1$	0.10	0.50

(j) Find the probability that $X+Y = 1$: $P(X+Y = 1)$.
(k) Find the expected value of X given that $Y = 1$: $E(X|Y=1)$.
(l) Find the variance of X: $\text{Var}(X)$.

Q6. (5 points) In the article "Filling the World's Belly" (The Economist, 2003) it is noted that 1/3 of adult Americans are obese. Thus in a room of ten random adult Americans, what is the probability that there are exactly five obese persons?

Q7. (5 points) State the central limit theorem and explain (in one or two sentences) its significance.

Q8. (5 points) Suppose that events *A* and *B* are mutually exclusive. Does this imply that they are independent? Explain.

Q9. (5 points) In the article "Does the US Budget Deficit Matter?" (BBC News, 2004) the report notes that the size of the US budget deficit is at a historically high level. Should large budget deficits, per se, be cause for concern? Explain? Moreover, if projections of future deficits are uncertain how should this affect the mandate at present to rein in spending?

Q10. (5 points) Suppose that the arrival time of a bus at a bus stop is uniformly distributed over a 60 minute range: $X \sim U[0,60]$. You arrive at the bus stop. What is the probability that a bus will come in the next 15 minutes?

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Bonus. (10 points) Consider two mutual funds, "Wall Street Boyz" and "Lombardi Street Girlz". You have W dollars to invest in these two funds. Your wealth is described by the function $W = \alpha X + (1-\alpha)Y$. Where αX is share of wealth in "Wall Street Boyz" and $(1-\alpha)Y$ is share of wealth placed in "Lombardi Street Girlz", and $\alpha \in [0,1]$. You are a very conservative investor, and your only objective is to minimise the variance of your portfolio. Suppose that:

(i) $\mathrm{Var}(X) = 1$; (ii) $\mathrm{Var}(Y) = 2$; (iii) $\mathrm{Cov}(X,Y) = -1$.

Find the value of α that minimises the variance of your portfolio.

Super Bonus. (5 points) Kazushi Sakuraba is a well renowned mixed martial arts fighter. He gained his fame by facing and defeating four members of the legendary Gracie family (at different times, of course!). In which dojo does Mr. Sakuraba train? And what is his professed style of fighting?

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