

# MPP Calculus Pre-Examination

This is not a mandatory examination. The intent of this mini-test is to assess and refresh your knowledge of college-level calculus. It should be very useful for everyone and help dictate at what level the course should be taught. The exam is very short and should not take you more than one hour to complete. Please write your solutions and either e-mail me your solutions (perhaps you can scan in your answers and send them to me as an e-mail attachment) or hand it in to me on the first day of class. If you are unsure of whether you are more suited for the algebra or calculus camp and would like some direction you should e-mail me your solutions ASAP so that I can give you some feedback. FYI, the questions are designed to be progressively harder as you proceed through each section and also as you move from section to section.

Kai Chan  
chan@princeton.edu

## 1 Functions

Graph the following functions:

1.  $f(x) = x^n$  for  $n = 1, 2, 3$
2.  $f(x) = 2x + 5$
3.  $f(x) = 3x^2 + 2x - 9$
4.  $f(x) = x/(1+x)$
5.  $f(x) = (\log x)/(1+e^x)$

## 2 Limits

Find the limits of the following functions:

1.  $\lim_{x \rightarrow -1} x^2 - 1$
2.  $\lim_{x \rightarrow 1} (x - 1)/(x^2 - 1)$
3.  $\lim_{x \rightarrow 0} (x^2 - x^3)/x$
4.  $\lim_{x \rightarrow 1} (x^2 - 1)/(e^x - 1)$
5.  $\lim_{x \rightarrow \infty} (\log x)/(1 - e^x)$

### 3 Definitions

Define the following terms:

1. Function.
2. Least upper bound.
3. Derivative of a function  $y = f(x)$ .
4. Tangent.
5. Fundamental Theorem of Calculus.

### 4 Derivatives

Calculate the derivatives of the following functions:

1.  $f(x) = ax + b$
2.  $f(x) = ax^n$
3.  $f(x) = xe^x - \log x$
4.  $f(x) = (1 + x) / (3 - 4x)$
5.  $f(x) = \log(1 + e^{x^2})$

### 5 Integrals

Calculate the following integrals:

1.  $\int x dx$
2.  $\int a(x + 3) dx$
3.  $\int_0^2 (x^3 + C) dx$
4.  $\int \log x dx$
5.  $\int xe^x dx$

## 6 Optimisation

Find the critical points of the following functions:

1.  $f(x) = x \quad x \in [0, 1]$

2.  $f(x) = x^2 \quad x \in (-1, 1)$

3.  $f(x) = x^3 - x^2 + 3x - 9 \quad x \in (-\infty, \infty)$

4.  $f(x) = \int_a^x (e^t + 3t^2 - 4t^3) dt \quad x \in (a, \infty)$

5.  $\max_{x_1, x_2} u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$  subject to the restriction  $p_1 x_1 + p_2 x_2 \leq I$

The mini-exam ends here. If you found this exam easy then you will coast through the calculus camp. If you found this exam hard, however, do not worry too much – it has probably been many years since you last saw calculus. I expect that most of you should find the first two questions in each section doable but that questions three to five are more challenging. In any case, you should expect to be able to know how to solve all of these questions after going through the math camp. Please feel free to get in touch with me (chan@princeton.edu) if you have any matters that you would like to discuss.

klc