ECO206Y Problem Set 2 (Second Half)

Due Thursday, July 18 (optional)

Question 1: n-firm oligopoly

Suppose there are n symmetrically identical firms competing under Cournot competition. Suppose that the market demand function is linear

$$p(y_1 + y_2 + \ldots + y_n) = a - b(y_1 + y_2 + \ldots + y_n) \qquad (1)$$

and that each firm faces a constant marginal cost of production

$$MC_1 = \ldots = MC_n = c: \qquad (2)$$

    (a) Find the reaction function for a typical firm.

    (b) Find the output and price combination chosen by a typical firm.

    (c) Now suppose that $n \to 1$. What happens to output and price?

Question 2: Duopoly with different cost structures

Suppose that a duopoly has a low- and high-cost firm, where

$$1 > MC_1(y_1) = C > c = MC_2(y_2) > 0, \qquad (3)$$

but that the firms are otherwise identical. The market demand is $p = 1 - y = 1 - y_1 - y_2$. Furthermore suppose that transfers are possible.

    (a) Find the Cournot level of output for each firm.

    (b) If the two firms decide to act collusively what output and price will be chosen. What range of payoffs (profits) does each firm receive?

    (c) Now suppose that the firms compete under Bertrand competition. What will be the equilibrium outputs and prices?

    (d) Now suppose that the firms decide to collude. What restriction on C is required for the high-cost firm to earn a non-zero profit under collusion?

Question 3: International duopolists

Suppose that there are two markets for maple syrup: Canada and Russia. Each country produces maple syrup (perfectly identical products). Originally markets are closed (i.e. there is no trade between the two countries) and in each country the maple syrup industry is served by a (domestic) monopolist. Now suppose that the countries open up to trade and that the cost of transporting maple syrup from one country to the other is t per unit of output. Let the (inverse) demand in Canada be given by

$$p = a - bz \qquad (4)$$

where $z = x + y$ is the quantity of maple syrup in the Canadian market. The quantity x is the amount of maple syrup supplied by the Canadian firm ("Jacque's Maple Syrup Company") and y is the amount of maple syrup supplied by the Russian company. In the Russian market the (inverse) demand function is

$$p^* = a^* - bz^* \qquad\qquad (5)$$

where $z^* = x^* + y^*$. The quantity x^* is the amount of output brought to the Russian market by the Canadian firm, and y^* is the quantity of output brought to market by the Russian firm. That is, the Canadian firm produces x (for Canada) and x^* (for Russia), while the Russian firm produces y (for Canada) and y^* (for Russia). Neither domestic monopolist has a fixed cost of production but faces a constant marginal cost of production equal to c per unit of output. Suppose that the two monopolists engage in Cournot competition in the Canadian market when trade is opened up. Find the reaction functions for each firm in the Canadian market. For what value of t will Canada and Russia engage in trade? What is the significance of this level of t that you found? Lastly, is trade beneficial when it happens?

Question 4: Games

(a) In the game rock-scissors-paper two players face off and each draw one of rock, scissors, or paper. The rules are that rock beats scissors, scissors beats paper, and paper beats rock (if both draw the same hand it is a tie). Suppose that two players are playing this game with a $1 stake on the game. Draw the payoff matrix for this game.

(b) A popular drinking game in East Asia is "ten". The object of the game is to call out a number $n \in \{0; 5; 10; 15; 20\}$ and for each player (usually two) to display zero, five, or ten fingers. Players take turns calling out numbers and the objective of the caller is to announce a number and have the sum of the two players' displayed fingers equal the number announced – fingers are displayed simultaneously as the caller announces a number. Thus the person who is not calling a number has the objective of not allowing the sum of displayed fingers equal the announcer's call. If the caller is correct in her count she calls again. When two consecutive counts match the caller's number she wins and her opponent chugs beer. (So your objective is to get your opponent wasted on cheap beer.) Suppose that player 1 has already called correctly once and is thus calling a second time. Draw the payoff matrix for this game where we suppose the payoff to the caller is $x \in \{\text{win}; \text{lose}\}$ and the payoff to the non-caller is $y \in \{\text{dry}; \text{chug}\}$. Eliminate all strategies for the caller which will never be played in equilibrium (assuming he is not drunk).

Question 5: Bertrand vs Cournot

Comment on the following statement:

> "The Bertrand model of oligopolies is inherently inferior to the Cournot model. This is because the Bertrand model predicts that in any oligopolistic market firms will charge a price equal to marginal cost. In the real world we seldom observe oligopolists charging a price equal to marginal cost; whereas the Cournot model predicts positive levels of profit. Moreover, the Cournot model has the more logical result that the oligopoly market structure tends towards perfect competition as the number of firms grows larger." (See question 1 or section 27.8 of your text.)

Question 6: Entry and deterrence

A monopolist faces an inverse demand function $p = 100 - y/2$ and a constant marginal cost of 40. A second firm is considering entering the market and if it enters, it too will face a constant marginal cost of 40. However, suppose that there is a one-time cost of entering the industry of $450. If and when the second firm decides to enter the two firms will engage in Cournot competition. Find the level of output that will deter the second firm from entering. Compare this level of output to the monopolist's level of output.